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Research Paper

Confidence Interval Estimation of Linear Model Parameter Ratios via Fieller's Method Under Heteroscedasticity Using Weighted Least Squares (Simulation Study)

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Abstract
Heteroskedasticity in linear models is a common problem that can cause parameter estimators to become inefficient and the estimation of the variance-covariance matrix to become inconsistent. This condition leads to incorrect and inaccurate statistical inference, including the construction of confidence intervals. Confidence interval estimation is not only performed for a single parameter but can also involve ratios, in which the Fieller method is commonly used for constructing confidence intervals for parameter ratios in linear models. This study examines the estimation of confidence intervals for parameter ratios using the Fieller method under heteroskedastic conditions by applying the Weighted Least Squares (WLS) method. Through simulations with variations in heteroskedasticity levels of $\gamma = 0,1,3,5$ and sample sizes of $n = 30$ and $n = 50$, the performance of the method was evaluated based on bias, Coverage Probability (CP), and Average Length (AL). The results show that the WLS method produces unbiased parameter estimators at all levels of heteroskedasticity. In addition, the Fieller method produces CP values around 0.95 with only small fluctuations and tends to yield shorter AL values. Therefore, the combination of the WLS and Fieller methods is proven to be stable and efficient for parameter estimation and confidence interval estimation of parameter ratios under heteroskedastic conditions.

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1. INTRODUCTION

A linear model is a model that describes the relationship between a dependent variable and independent variables, expressed through linear parameters. Generally, a linear model is expressed as $y = X\beta + \varepsilon$, where y is the observed random variable vector, X is the matrix of explanatory variables, β is the vector of parameters whose values are unknown, and ε is the error vector assumed to be normally distributed with $E(\varepsilon) = 0$ and $\mathrm{var}(\varepsilon) = \Sigma$ [1, 2]. Linear regression analysis relies on certain assumptions, one of which is homogeneity of variance [3]. If the classical assumptions are met, parameter estimation can be performed using the Least Squares method, provided that the errors have a mean of zero and constant variance [4].

In inference statistics, researchers are often interested in constructing confidence intervals that contain the true parameter with a high probability. A good confidence interval has a high coverage probability and a relatively short average length [5]. The coverage probability is a measure that indicates the long-term percentage of the time a confidence interval contains the unknown parameter. An interval is considered good if its coverage probability value is close to the specified nominal level [6]. Conceptually, this measure represents the probability that the lower and upper bounds of the interval encompass the true parameter. Therefore, coverage probability is used as the primary indicator in evaluating the validity of confidence intervals [7, 8]. An interval is called conservative if its coverage probability is greater than the specified confidence level, and liberal if it is less than that level. If more than one interval has good control and similar coverage probabilities, then the interval with the shorter width is preferred [9]. On the other hand, width or average length is used to measure the precision of the point estimator. A narrower interval indicates a higher level of precision [10].

One particularly difficult form of confidence interval estimate is when the parameter under study is a ratio. One of the popular method is the Fieller method. The Fieller method is a standard procedure for constructing confidence intervals for ratio statistics of several normally distributed parameters and has been widely used in biological, health, and economic research [11]. In statistics, Fieller's theorem is used to construct a confidence region for the ratio of two normal population means [12]. This method works by expressing the ratio as a linear combination of two random variables, thereby simplifying the construction of the confidence interval for the ratio [13].

In addition to the Fieller method, another commonly used approach is the Delta method. However, several studies have demonstrated that the Fieller method performs better than the Delta method. Previous research has shown that the Fieller method provides better coverage probabilities and more accurate evaluations of the precision and power of ratio tests through simulation studies [14]. Moreover, the Fieller method is more capable of handling situations in which the denominator approaches zero and the ratio distribution is asymmetric, whereas the Delta method tends to produce biased intervals under such conditions [15]. The Fieller's method is also considered robust and able to provide more accurate information regarding estimation uncertainty, even yielding unbounded intervals for ratio parameters that are susceptible to identification problems and irregular distributions [16]. In a study on insecticide dosages, the Fieller method achieved a high coverage probability of up to 95% [17]. Owing to these advantages, the Fieller method has been widely applied in linear model analysis [18]. Previous studies have also applied this method to linear models using drug dosage data; however, they still assumed that the error variance is constant [19].

heteroscedasticity, is often encountered. Heteroscedasticity causes the ordinary least squares estimator no longer have minimum variance, although it remains linear and unbiased. Furthermore, this condition makes the standard error calculation in the OLS method unreliable [20]. One method that can be used to address this issue is Weighted Least Squares (WLS), which uses weights proportional to the inverse of the response variable's variance [20]. This method accounts for heteroscedasticity, ensuring that parameter estimates remain consistent, efficient, and unbiased within the Best Linear Unbiased Estimator (BLUE) framework [22, 22]. With the application of WLS, parameter estimators become more efficient, so it is expected that the formation of confidence intervals will be more accurate. Study by Nisa et al. [23] shows that the WLS method is capable of producing consistent estimators under conditions of heteroscedasticity. Other studies also indicate that the Robust Method of Moments WLS is effective in addressing heteroscedasticity and outliers in regression models, thereby improving the quality of regression parameter estimates [24]. In addition, this method is also used in robust regression with weights that depend on the magnitude of the residuals [25].

If heteroscedasticity is not addressed, the parameter estimator becomes inefficient, resulting in less accurate statistical inferences, including the construction of confidence intervals. This inefficiency affects the calculation of the standard error, causing the t-test to become unstable and potentially leading to bias in the analysis [26]. Therefore, if heteroscedasticity is not addressed, the confidence intervals constructed using the Fieller method may have a coverage probability that deviates from the nominal confidence level due to the use of an inconsistent covariance matrix estimate.

To the present, research on the construction of confidence intervals for parameter ratios using the Fieller method in linear models with heteroscedasticity via a simulation approach remains relatively limited. Some previous studies, such as those by Hirschberg & Lye [11], Freedman [14], and Zerbe [19], generally consider only models with constant error variance. Therefore, this study was conducted to fill this gap by examining confidence intervals for parameter ratios using the Fieller method in linear models with heteroscedasticity addressed by WLS through a Monte Carlo simulation approach.

2. METHODS

This study employs a simulation approach to evaluate the performance of the Fieller method in constructing confidence intervals for parameter ratios in linear models under heteroscedastic conditions. To address heteroscedasticity, the Weighted Least Squares (WLS) method is applied. The data used in this study are artificially generated based on a multiple linear regression model with three independent variables. The linear regression model used in this study is expressed in Equation (1) as follows:

$$y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i, \quad i = 1, 2, \ldots, n \qquad (1)$$

with $X_{ij} \sim U[1, 4]$ for $j = 1, 2, 3$ and each variable is mutually independent. The true parameter values in the simulation are set as $\beta_0 = 0.5$, $\beta_1 = 1$, $\beta_2 = 1.5$, and $\beta_3 = 2$.

2.1 Formation of Heteroscedasticity

In regression analysis practice, the assumption of constant variance is often violated, leading to inefficient estimators. In this study, the error term is modeled as in

$$\varepsilon_i \sim N(0, \sigma^2 V)$$

where $V = diag(v_1, v_2, \ldots, v_n)$, with

$$v_i = z(\gamma)(|X_{i1}| + |X_{i2}| + |X_{i3}|)^\gamma$$

The parameter $\gamma = 0, 1, 3, 5$ is used to control the level of heteroscedasticity. The scaling factor $z(\gamma)$ is defined such that the average error variance equals one, which is shown in Equation (2).

$$z(\gamma) = \frac{n}{\sum_{i=1}^{n}(|X_{i1}| + |X_{i2}| + |X_{i3}|)^{\gamma}} \tag{2}$$

This approach is used to ensure that the average variance remains constant, allowing for fair comparisons across conditions. The heteroscedasticity design refers to the study by Sterchi and Wolf [27]. Verification of heteroscedasticity is carried out using the ratio which is given in Equation (3).

$$\text{Heteroscedasticity Ratio} = \frac{\max(v(x))}{\min(v(x))} \tag{3}$$

A ratio value of 1 indicates homoscedasticity, while a value greater than 1 indicates the presence of heteroscedasticity.

2.2 Parameter Estimation using Weighted Least Squares (WLS)

The WLS method is a special case of the Generalized Least Squares method when the error variance–covariance matrix is diagonal, namely:

$$V = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n^2 \end{bmatrix}$$

The underlying assumption states that $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ are mutually independent but have unequal variances [4]. In the general linear model under WLS, it is assumed that $E(\varepsilon) = 0$ and $\text{var}(\varepsilon) = \sigma^2 V$. The parameter estimator using the WLS method is given by Equation (4).

$$\beta = \left(X^{\mathrm{T}} V^{-1} X\right)^{-1} X^{\mathrm{T}} V^{-1} y \tag{4}$$

The estimator of the error variance is expressed in Equation (5).

$$s^2 = \frac{(\mathbf{y} - X\hat{\beta}^*)' V^{-1} (\mathbf{y} - X\hat{\beta}^*)}{n - p} \tag{5}$$

These estimators are known to be the Best Linear Unbiased Estimators (BLUE).

2.3 Fieller Method for Parameter Ratio Confidence Intervals

The Fieller's method is used to construct confidence intervals for the ratio of two parameters, each of which is normally distributed [10]. This approach is carried out by transforming the parameter ratio into a linear function of two variables assumed to follow a bivariate normal distribution [28]. The confidence interval in this study is constructed based on parameter estimators obtained from the WLS method. Let the parameter ratio be expressed as presented in Equation (6).

$$\rho_{WLS} = \frac{\mathbf{K}'\hat{\beta}_{WLS}}{\mathbf{L}'\hat{\beta}_{WLS}} \tag{6}$$

Following the procedure of study Daoud [29] based on the weighting from the WLS method, the test statistic for the linear ratio of the model parameters is obtained and denoted as T as in Equation (7).

$$T = \frac{\mathbf{K}'\hat{\beta}_{WLS} - \rho\,\mathbf{L}'\hat{\beta}_{WLS}}{\hat{\sigma}\sqrt{\begin{aligned}&\mathbf{K}'(X'V^{-1}X)^{-1}\mathbf{K} \\ &- 2\rho\,\mathbf{K}'(X'V^{-1}X)^{-1}\mathbf{L} \\ &+ \rho^2\mathbf{L}'(X'V^{-1}X)^{-1}\mathbf{L}\end{aligned}}} \tag{7}$$

Following a t-student distribution with $n - p$ degrees of freedom, a $100(1-\alpha)\%$ confidence interval for ρ is obtained using the Fieller approach. In probabilistic terms, the probability that T lies within the interval $[-t, t]$ is equivalent to satisfying the quadratic inequality as in Equation (8) as follows:

$$1 - \alpha = P[t \le T \le t] = P(A\rho^2 + B\rho + C \le 0) \tag{8}$$

with:

$$\begin{aligned} A &= (\mathbf{L}'\hat{\beta}_{WLS})^2 - t^2\mathbf{L}'(X'V^{-1}X)^{-1}\mathbf{L}\sigma^2, \\ B &= 2\left[t^2\mathbf{K}'(X'V^{-1}X)^{-1}\mathbf{L}\sigma^2 - (\mathbf{K}'\hat{\beta}_{WLS})(\mathbf{L}'\hat{\beta}_{WLS})\right], \\ C &= (\mathbf{K}'\hat{\beta}_{WLS})^2 - t^2\mathbf{K}'(X'V^{-1}X)^{-1}\mathbf{K}\sigma^2. \end{aligned}$$

Then a, b and c denote the observed values of A, B, and C. Then the $100(1-\alpha)\%$ confidence interval for ρ is given by

$$\left[\frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right]$$

with the condition $a > 0$ and $b^2 - 4ac > 0$ [29]. In this study, the vectors $\mathbf{K}'$ and $\mathbf{L}'$ used are as shown in Table 1.

Table 1. Constant Vectors K′ and L′

K′	L′
[0 1 0 0]	[0 0 1 0]
[0 0 0 1]	[0 0 1 0]
[0 1 3 0]	[0 0 0 1]
[0 4 −2 3]	[0 1 1 1]

2.4 Performance Measurement of the Method

The evaluation of the method is conducted by assessing how well the applied approach performs. The performance of the WLS method is measured using: $bias(\hat{\theta}) = |\hat{\theta} - \theta|$

The evaluation of the Fieller method is carried out using Coverage Probability (CP) and Average Length (AL). Coverage probability represents the proportion of confidence intervals that contain the true parameter in the long run [4]. The formula for CP is given by Equation (9) as follows:

$$\text{CP} = \frac{\sum_{i=1}^{m}[\rho \in I_i]}{m} \tag{9}$$

Average length measures the average width of the confidence intervals and represents the precision of the estimator [5]. The formula for AL is given by Equation (10) below:

$$AL = \sum_{i=1}^{m} \frac{length\ I_i}{m} \tag{10}$$

where I_i denotes the i-th interval and m is the number of replications.

2.5 Simulation Procedures

The simulation was conducted with 10,000 replications for each condition. The research procedure is as follows:

1. Generating independent variables $X_{ij} \sim U[1,4]$, for $i = 1,2,\ldots,n$ and $j = 1,2,3$.
2. Setting the true parameters $\beta_0 = 0.5$, $\beta_1 = 1$, $\beta_2 = 1.5$, $\beta_3 = 2$ and $\sigma^2 = 1$.
3. Constructing heteroscedastic error variances using parameter γ.
4. Estimating the parameters $\hat{\beta}$ and $\hat{\sigma}^2$ using WLS.
5. Calculating the ratio value ρ based on vectors $\mathbf{K}'$ and $\mathbf{L}'$.
6. Computing the estimated value $\hat{\rho}$ from β_{WLS} estimation result then calculating the bias for each predetermined ρ_i.
7. Constructing confidence intervals using the Fieller method for each ρ_i.

Calculating the CP and AL for each interval to evaluate the performance of the Fieller method at each level of heteroscedasticity.

3. RESULT AND DISCUSSIONS

3.1 Performance Measurement of the Method

The level of heteroskedasticity is measured using the formula in Equation (1), yielding the following results shown in Table 2.

The variance ratio values increase as the value of γ increases. At $\gamma = 0$, the variance ratio equals 1, indicating a homoscedastic condition, whereas at $\gamma > 0$ the variance increases, indicating the presence of heteroskedasticity. In addition, for the same value of γ, the sample size $n = 50$ produces a larger variance ratio than $n = 30$. These results indicate that the simulation scenarios developed are able to represent various levels of heteroskedasticity in accordance with the research design.

Table 2. Level of Heteroskedasticity (γ) for Each Sample Size

	Level of Heteroskedasticity			
n/γ	0	1	3	5
30	1.00	2.5191166	15.9861844	101.4475037
50	1.00	2.7102129	19.9072012	146.2234442

3.2 Parameter Estimation using Weighted Least Squares (WLS)

Parameter estimation was performed using the Weighted Least Squares (WLS) method based on Equations (4) and (5). The simulation results show that the estimated parameters are close to the true parameter values. The following presents the parameter bias obtained using the WLS method.

Table 3. Bias Estimation of Parameters β and σ^2

n	γ	β_0	β_1	β_2	β_3	$\hat{\sigma}^2$
30	0	0.011	0.00006	0.0046	0.00046	0.0009
30	1	0.013	0.00003	0.0050	0.00084	0.0009
30	3	0.014	0.00006	0.0052	0.00127	0.0009
30	5	0.011	0.00035	0.0043	0.00123	0.0008
50	0	0.003	0.00045	0.0012	0.00123	0.0022
50	1	0.002	0.00046	0.0011	0.00003	0.0022
50	3	0.001	0.00033	0.0008	0.00019	0.0022
50	5	0.000	0.00007	0.0005	0.00025	0.0024

The bias values of all parameters at various levels of heteroscedasticity are relatively small and close to zero (Table 3). This indicates that the WLS method is able to produce unbiased estimators under all heteroscedasticity conditions. In addition, the sample size $n = 50$ tends to produce smaller bias values compared to $n = 30$, indicating that an increase in sample size can improve the accuracy of parameter estimation.

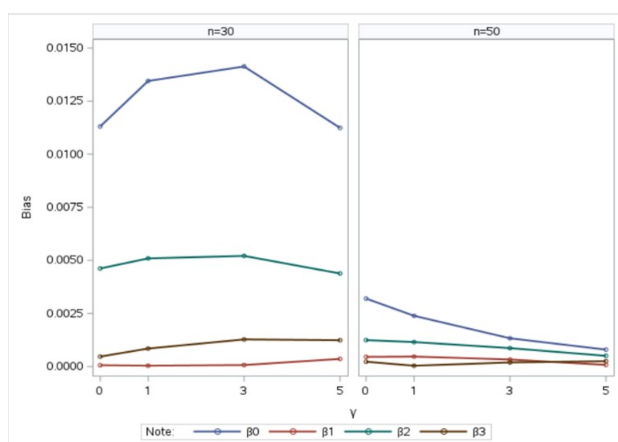

Figure 1. Bias of Parameter β for Each Sample

Figure 1 shows the visualization of parameter bias that the bias patterns of all parameters are relatively stable across various levels of heteroscedasticity. The bias for the sample size $n = 30$ appears to fluctuate more compared to $n = 50$, whereas the larger sample size produces smaller and more stable bias values. This indicates that parameter estimation improves as the sample size increases.

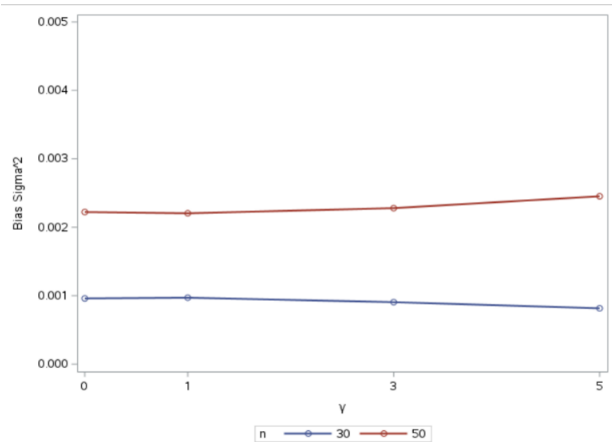

Figure 2. Bias of Parameter σ^2 for Each Sample

Figure 2 shows the visualization of the bias of σ^2 that the bias values for both sample sizes are relatively small and stable across various levels of heteroscedasticity. For the sample size $n = 30$, the bias of σ^2 tends to decrease slightly as the value of γ increases, whereas for $n = 50$ the bias tends to increase slightly. However, these changes are not very significant, indicating that the estimation of σ^2 remains stable under all heteroscedasticity conditions.

3.3 Estimation of Parameter Ratios

The parameter ratio of interest in this study is expressed as in Equation 4. The ratio is estimated using a WLS-based estimator. The simulation results show that the estimated ratio values are close to the true value with relatively small bias. The bias of the ratio is presented in Table 4.

Table 4. Estimation of Parameter Ratios

n	γ	ρ_1	ρ_2	ρ_3	ρ_4
30	0	0.0070	0.0250	0.0299	0.0053
30	1	0.0076	0.0245	0.0342	0.0041
30	3	0.0063	0.0202	0.0366	0.0025
30	5	0.0032	0.0142	0.0304	0.0022
50	0	0.0043	0.0140	0.0233	0.0026
50	1	0.0041	0.0140	0.0234	0.0025
50	3	0.0023	0.0117	0.0197	0.0020
50	5	0.0002	0.0079	0.0138	0.0012

The bias values of the parameter ratios ρ_1, ρ_2, ρ_3, and ρ_4 under all conditions are relatively small and close to zero. This indicates that the WLS method is able to produce unbiased ratio estimators, even at high levels of heteroscedasticity. In addition, increasing the sample size from $n = 30$ to $n = 50$ results in smaller bias values, indicating an improvement in the accuracy of parameter ratio estimation.

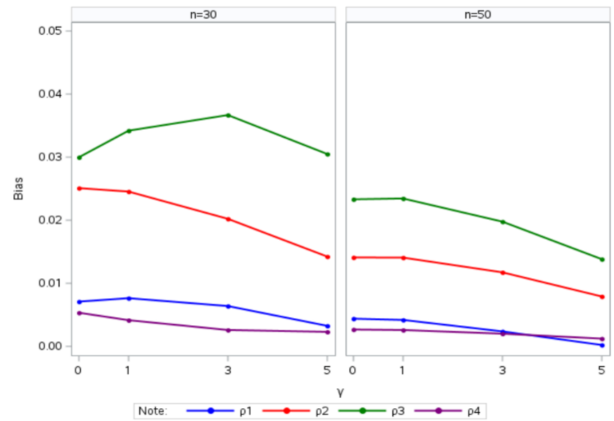

Figure 3. Bias of Parameter ρ for Each Sample

Figure 3 displays the visualization of the parameter ratio bias that the bias values tend to decrease as the value of γ increases, particularly for the sample size $n = 50$. For the sample size $n = 30$, some parameters still exhibit fluctuations, especially ρ_3, whereas for the sample size $n = 50$ the pattern of bias changes appears more stable (Figure 3). This indicates that the estimation of parameter ratios becomes better and more stable with larger sample sizes.

3.4 Fieller's Confidence Interval

The confidence intervals for the parameter ratio are constructed using the Fieller method. The performance of the confidence intervals is evaluated using CP and AL based on Equations (9) and 10. The simulation results are presented in Table5.

CP values are around 95% under all conditions, indicating that the Fieller method is able to maintain the expected confidence level. In addition, the AL tends to decrease as the level of heteroscedasticity increases. This is influenced by the weighting in the WLS method, which is affected by the scaling factor $z(\gamma)$. Furthermore, increasing the sample size from $n = 30$ to $n = 50$ also reduces the interval length due to the decrease in variance estimation.

The visualization of CP for the sample size $n = 30$ shows that the CP values of all parameter ratios are relatively stable and remain around 0.95 across various levels of heteroscedasticity which is shown in Figure 4. Some parameters exhibit slight fluctuations as the value of γ increases; however, these changes are not very significant. This indicates that the method used is able to maintain a good interval confidence level despite the presence of heteroscedasticity.

The visualization of CP for the sample size $n = 50$ shows that the CP values of all parameter ratios are relatively stable and tend to be closer to the nominal confidence level of 0.95

Table 5. Simulation Results of Confidence Intervals for Each Ratio

n	ρ_i	γ	95% Confidence Interval	AL	CP	n	ρ_i	γ	95% Confidence Interval	AL	CP
30	ρ_1	0	[0.38113−1.11749]	0.73635	0.9519	50	ρ_1	0	[0.42984−0.99493]	0.56508	0.9491
		1	[0.38921−1.11492]	0.72570	0.9521			1	[0.43277−0.98883]	0.55606	0.9481
		3	[0.41466−1.06345]	0.64879	0.9524			3	[0.45851−0.94004]	0.48153	0.9489
		5	[0.44660−0.98414]	0.53753	0.9530			5	[0.50426−0.86347]	0.35920	0.9486
30	ρ_2	0	[0.98369−2.05545]	1.07176	0.9519	50	ρ_2	0	[1.04142−1.78237]	0.74095	0.9495
		1	[0.97710−2.05008]	1.07297	0.9509			1	[1.04231−1.78030]	0.73799	0.9495
		3	[0.98449−1.96423]	0.97974	0.9512			3	[1.06551−1.72839]	0.66287	0.9480
		5	[1.01800−1.83777]	0.81976	0.9495			5	[1.10779−1.64195]	0.53416	0.9476
30	ρ_3	0	[2.34815−4.43280]	2.08464	0.9522	50	ρ_3	0	[2.57802−4.15423]	1.57621	0.9487
		1	[2.35709−4.46979]	2.11270	0.9514			1	[2.57584−4.15456]	1.57871	0.9470
		3	[2.41613−4.43998]	2.02384	0.9502			3	[2.61602−4.07357]	1.45754	0.9485
		5	[2.51186−4.28266]	1.77080	0.9420			5	[2.69800−3.93223]	1.23423	0.9471
30	ρ_4	0	[1.80771−2.67268]	0.86496	0.9502	50	ρ_4	0	[1.88094−2.58188]	0.70094	0.9509
		1	[1.80889−2.65688]	0.84798	0.9506			1	[1.88522−2.57717]	0.69195	0.9499
		3	[1.83783−2.60980]	0.77196	0.9519			3	[1.92360−2.53493]	0.61133	0.9501
		5	[1.89165−2.55436]	0.66271	0.9508			5	[1.98977−2.46363]	0.47386	0.9502

Note:
CP = Coverage Probability
AL = Average Length

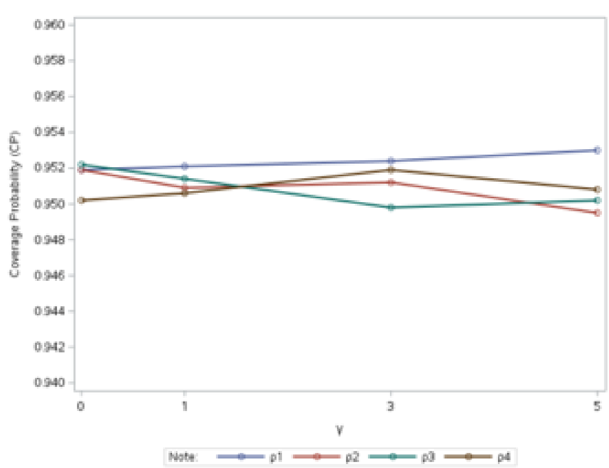

Figure 4. Coverage Probability Plot for Sample Size 30

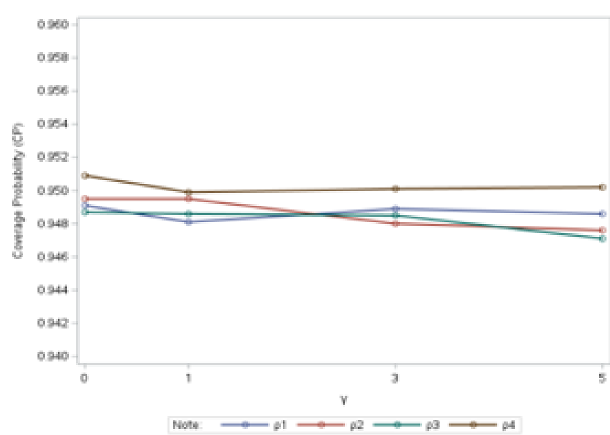

Figure 5. Coverage Probability Plot for Sample Size 50

across various levels of heteroscedasticity which is shown in Figure 5. The fluctuations observed for each parameter are also smaller compared to the sample size $n = 30$. This indicates that increasing the sample size produces more accurate and stable confidence interval estimations.

Figure 6 displays the visualization of AL for the sample size $n = 30$ that the confidence interval lengths tend to decrease as the value of γ increases for all parameter ratios (Figure 6). Although slight fluctuations are observed in some parameters, the decreasing pattern appears fairly consistent. This indicates that the resulting confidence intervals become narrower at higher levels of heteroscedasticity.

The visualization of AL for the sample size $n = 50$ shows that the confidence interval lengths tend to decrease consistently as the value of γ increases for all parameter ratios as shown in Figure 7. Compared to the sample size $n = 30$, the average length values for $n = 50$ are also smaller, resulting in narrower and more efficient confidence intervals.

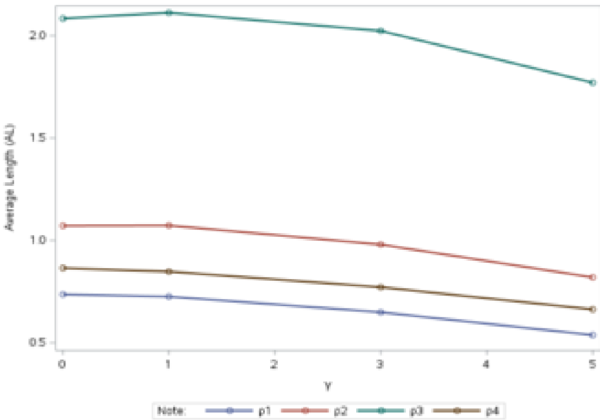

Figure 6. Coverage Probability Plot for Sample Size 50

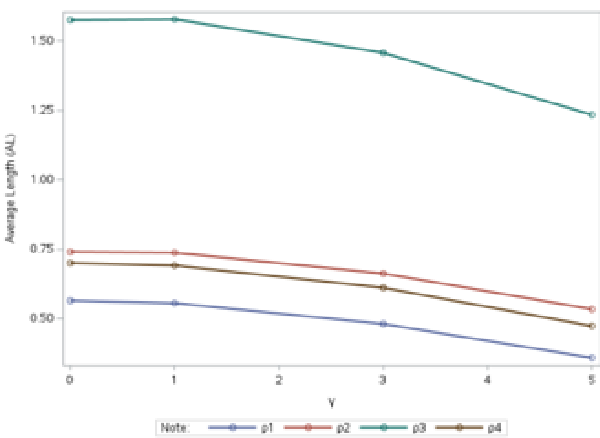

Figure 7. Average Length Plot for Sample Size 50

4. CONCLUSIONS

The results show that the Weighted Least Squares (WLS) method is able to address heteroscedasticity by producing unbiased parameter estimators. This is supported by bias values that are close to zero for all parameters and estimated ratios. In addition, the Fieller method for constructing confidence intervals of the parameter ratio demonstrates good performance, indicated by Coverage Probability values that consistently remain around the nominal confidence level and decreasing interval lengths. These findings indicate that the use of weighting through WLS helps support the stability and accuracy of the confidence intervals constructed using the Fieller method, even under heteroskedastic conditions. Therefore, the combination of the WLS and Fieller methods is an effective approach for inference on parameter ratios in regression models with heteroscedasticity.

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