Integra: Journal of Integrated Mathematics and Computer Science 3(1) 6-15

Integra: Journal of Integrated Mathematics and Computer Science

Journal Homepage: https://integrajimcs.com/index.php/integra/index

integra

Journal of Integrated Mathematics and Computer Science

Research Paper

Note for Soft MultiExpert Graph and MultiSoft MultiExpert Graph

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Keywords

Soft Graph, Soft Expert Graph, Soft Multi Expert Graph, Multi Soft Multi Expert Graph, Multi Soft Graft

Abstract

This paper studies multilevel extensions of soft-set-based graph models for uncertainty-aware decision support. We first recall soft, multisoft, soft expert, and multisoft multi-expert sets, which encode parameterized and expert-dependent approximations of a universe. Building on these notions, we introduce the MultiSoft Graph, a family of induced subgraphs of a given graph indexed by multilevel parameter blocks, and show that it strictly generalizes classical soft graphs while inducing a canonical multisoft set on the vertex set. We then define Soft MultiExpert Graphs and MultiSoft MultiExpert Graphs, providing a unified framework that jointly handles graph topology, multiple parameters, and expert opinions.

Received: 5 December 2025, Accepted: 1 March 2026

https://doi.org/10.26554/integra.20263146

1. INTRODUCTION

A Soft Set offers a parameter-based representation of uncertainty: each parameter (or attribute) is associated with a subset of a given universe, so that vague or incomplete information can be encoded in a simple and flexible way [1]. Over the years, many refinements and variants of this framework have been introduced, including the HyperSoft Set [2, 3], MultiSoft Set [4, 5], Fuzzy Soft Set [6], Neutrosophic Soft Set [7, 8], SuperHyperSoft Set [9, 10, 11, 12], ContraSoft Set [13], Soft Expert Set [14, 15], TreeSoft Set [16, 17], and ForestSoft Set [18, 19]. Taken together, these structures form an important class of modeling tools that are actively used in decisionmaking and related application areas.

Soft Graphs extend the soft-set philosophy from sets to graphs by assigning to each parameter a subgraph of a fixed underlying graph, thereby modeling parameter-dependent selections of vertices and edges under uncertainty and imprecision [20, 21]. Further generalizations have been developed by embedding Soft Graphs into richer combinatorial frameworks such as HyperGraphs [22, 23, 24] and SuperHyperGraphs [25, 26]. These graph-based soft structures are designed for decision-making on real-world networks in which data are incomplete, noisy, or context dependent, and a substantial body of work has been devoted to their analysis. Among the extended notions, a Soft Expert Graph has been proposed as a refinement of Soft Graphs: it assigns induced subgraphs to triples consisting of a parameter, an expert, and a binary opinion, thus aggregating expert-labeled structural views into a single graphical framework.

In light of the above developments, research on Soft Sets and Soft Expert Sets is clearly significant; however, more complex constructions such as MultiSoft MultiExpert Sets have not yet been investigated exhaustively. To address this gap, we introduce the notion of a MultiSoft Graph, defined as a family of induced subgraphs of a given graph indexed by multilevel parameter blocks. We prove that MultiSoft Graphs properly extend classical Soft Graphs and, at the same time, canonically induce an underlying MultiSoft Set on the vertex set. Building on this, we define Soft MultiExpert Graphs and MultiSoft MultiExpert Graphs, thereby providing a unified framework that simultaneously incorporates graph topology, multiple interacting parameters, and expert opinions within a single coherent formalism.

2. PRELIMINERIES

Throughout this paper we work only with finite sets. In this section we recall the basic notions and notation that will be used in the sequel.

2.1 Soft Set and Multi Soft Set

A soft set on a universe U is a parameter–indexed family of subsets of U, describing which elements of U approximately satisfy each parameter [1, 27]. A multisoft set is a refinement of this idea in which several pairwise disjoint collections of parameters are allowed, and each admissible parameter combination is associated with a subset of U [5, 10, 28].

In a soft expert set, one additionally records the expert and the corresponding opinion, so that parameter–expert–opinion triples are mapped to subsets of U; equivalently, one obtains a parameterized mapping from $E \times X \times O$ into $\mathcal{P}(U)$ [7, 29]. For completeness, we recall formal definitions of these notions below [30, 31].

Definition 2.1 *Soft Set [1, 27] Let U be a universe and E a set of parameters. A soft set over U is a pair (F, E) where*

$$F : E \longrightarrow \mathcal{P}(U)$$

is a mapping. For each parameter $e \in E$, the subset $F(e) \subseteq U$ consists of those elements of U that are regarded as approximately satisfying the parameter e.

Definition 2.2 (Soft Expert Set [7? , 29, 32]) *Let U be a universe, E a set of parameters, X a set of experts, and $O = \{0, 1\}$ a set of opinions. Put*

$$Z := E \times X \times O$$

and let $A \subseteq Z$ be nonempty. A soft expert set on U is a pair (G, A) where

$$G : A \longrightarrow \mathcal{P}(U)$$

assigns to each triple $\alpha = (e, x, o) \in A$ a subset $G(\alpha) \subseteq U$. Thus $G(\alpha)$ collects the elements of U supported by expert x, with opinion o, under parameter e.

Example 2.1 (Soft Expert Set) *Let*

$$U = \{u_1, u_2, u_3\}$$

be three candidate smartphones. Let the parameter set be

$$E = \{e_1, e_2\},$$

where e_1 stands for "high battery life" and e_2 for "low price". Let

$$X = \{x_1, x_2\}$$

be two experts and

$$O = \{1, 0\},$$

where 1 means "agree" and 0 means "disagree".

Set

$$Z = E \times X \times O$$

and take

$$A = \{(e_1, x_1, 1), (e_2, x_1, 1), (e_1, x_2, 1)\} \subseteq Z.$$

Define

$$G : A \longrightarrow \mathcal{P}(U)$$

by

$$G(e_1, x_1, 1) = \{u_1, u_2\}, \quad G(e_2, x_1, 1) = \{u_1\}, \quad G(e_1, x_2, 1) = \{u_2, u_3\}.$$

Then (G, A) is a soft expert set on U. For instance, $G(e_1, x_1, 1)$ is the set of smartphones that expert x_1 considers to have high battery life, and $G(e_2, x_1, 1)$ is the set that the same expert considers sufficiently cheap.

Definition 2.3 (Multisoft Set [5, 10, 28]) *Let U be a nonempty universe. Let $\{E_i\}_{i=1}^{n}$ be pairwise disjoint parameter sets and define*

$$E = \bigcup_{i=1}^{n} E_i.$$

Let $A \subseteq \mathcal{P}(E)$ be a nonempty family of parameter subsets. A multisoft set over U is a pair (H, A) where

$$H : A \longrightarrow \mathcal{P}(U)$$

is a mapping. For each $a \in A$, the set $H(a) \subseteq U$ is called the approximate value of the parameter subset a.

Example 2.2 (Multisoft Set) *Let*

$$U = \{c_1, c_2, c_3, c_4\}$$

be four customers. Consider two disjoint parameter sets

$$E_1 = \{p_{low}, p_{high}\}, \quad E_2 = \{q_{basic}, q_{premium}\},$$

where E_1 describes price level (low / high) and E_2 service level (basic / premium). Put

$$E = E_1 \cup E_2 = \{p_{low}, p_{high}, q_{basic}, q_{premium}\}.$$

Define

$$A = \{a_1, a_2\} \subseteq \mathcal{P}(E),$$

where

$$a_1 = \{p_{low}, q_{basic}\}, \quad a_2 = \{p_{high}, q_{premium}\}.$$

Set

$$H : A \longrightarrow \mathcal{P}(U)$$

by

$$H(a_1) = \{c_1, c_2\}, \quad H(a_2) = \{c_3, c_4\}.$$

Then (H, A) is a multisoft set over U, where $H(a_1)$ is the approximate set of customers preferring low price and basic service, while $H(a_2)$ collects those roughly preferring high price and premium service.

2.2 Soft Multi Expert Set

Informally, a Soft MultiExpert Set assigns to every parameter a finite multiset of expert–opinion entries, each entry carrying its own approximate subset of the universe [33].

Definition 2.4 (Soft MultiExpert Set [33]) *Let U be a universe, E a set of parameters, X a set of experts, and $O = \{0, 1\}$ a set of binary opinions.*

For any set Y, write

$$\mathcal{M}(Y) = \{m : Y \to \mathbb{N}_0 \mid \operatorname{supp}(m) \text{ is finite}\}$$

for the collection of finite multisets on Y, where

$$\operatorname{supp}(m) := \{y \in Y : m(y) > 0\}.$$

A Soft MultiExpert Set over U is a pair (F, E) such that

$$F : E \longrightarrow \mathcal{M}(X \times O \times \mathcal{P}(U))$$

is a mapping.

For each parameter $e \in E$, the multiset $F(e)$ consists of triples

$$(x, o, A_{x,o}) \quad (x \in X,\ o \in O,\ A_{x,o} \subseteq U),$$

possibly with repetitions, and the intended meaning is that "expert x, having opinion o, supports the approximate subset $A_{x,o}$" under parameter e.

and for each $e \in E$ define a finite multiset $F(e) \in \mathcal{M}(Y)$ by specifying its nonzero multiplicities.

For the parameter e_{battery} set

$$F(e_{\text{battery}})(x_1, 1, \{u_1, u_2\}) = 2, \tag{1}$$

$$F(e_{\text{battery}})(x_2, 1, \{u_1\}) = 1, \tag{2}$$

and

$$F(e_{\text{battery}})(y) = 0 \quad \text{for all other } y \in Y.$$

Thus expert x_1 strongly (multiplicity 2) supports $\{u_1, u_2\}$ as having long battery life, while expert x_2 weakly supports $\{u_1\}$.

For the parameter e_{weight} set

$$F(e_{\text{weight}})(x_1, 1, \{u_2, u_3\}) = 1, \tag{3}$$

$$F(e_{\text{weight}})(x_2, 1, \{u_2, u_3\}) = 2, \tag{4}$$

and

$$F(e_{\text{weight}})(y) = 0 \quad \text{otherwise.}$$

Then (F, E) is a Soft MultiExpert Set on U, since each parameter is assigned a finite multiset of expert–opinion–subset triples.

2.3 MultiSoft Multi Expert Set

A MultiSoft MultiExpert Set combines the multilevel parameter structure of a multisoft set with multisets of parameter–expert–opinion subsets, each linked to a subset of the universe [33].

Definition 2.5 (MultiSoft MultiExpert Set) *Let U be a universe, E a set of parameters, X a set of experts, and $O = \{0, 1\}$ a set of binary opinions. Set*

$$Z := E \times X \times O.$$

Consider the class of finite multisets of subsets of Z,

$$\mathcal{M}(\mathcal{P}(Z)) := \{A : \mathcal{P}(Z) \to \mathbb{N}_0 \mid \operatorname{supp}(A) \text{ is finite}\}.$$

A MultiSoft MultiExpert Set over U is a pair (F, A), where $A \in \mathcal{M}(\mathcal{P}(Z))$ and

$$F : \mathcal{P}(Z) \longrightarrow \mathcal{P}(U)$$

is a mapping with the property that, whenever $\alpha \subseteq Z$ satisfies $A(\alpha) > 0$, the set $F(\alpha) \subseteq U$ is the approximate value associated with the parameter–expert–opinion subset α.

2.4 Soft Graph

A Soft Graph assigns to each parameter an induced vertex–subgraph of a given graph, thereby producing a parameterized family of subgraphs [34]. Related structures include Neutrosophic Soft Graphs [35], HyperSoft Graphs [36], and other soft-set–based graph extensions.

Definition 2.6 (Soft Graph) *Let $G = (V, E)$ be a (finite) simple graph, where V is the vertex set and*

$$E \subseteq \big\{\{u, v\} : u, v \in V,\ u \neq v\big\}$$

is the edge set. Let P be a nonempty set of parameters and let $A \subseteq P$ be a nonempty index set.

A mapping

$$F : A \longrightarrow \mathcal{P}(V)$$

is called a soft vertex assignment on G. For each $a \in A$ we define

$$V_a := F(a) \subseteq V,$$

$$E_a := \big\{\{u, v\} \in E : u, v \in V_a\big\},$$

and denote the corresponding subgraph by

$$G_a := (V_a, E_a).$$

The triple

$$SG = \big(G, (F, A)\big)$$

is called a Soft Graph over G if, for every $a \in A$, the graph $G_a = (V_a, E_a)$ is a vertex-induced subgraph of G. Equivalently, a Soft Graph is a parameterized family $\{G_a : a \in A\}$ of induced subgraphs of G indexed by the parameter set A.

2.5 Soft Expert Graph

A Soft Expert Graph assigns induced subgraphs using triples of parameter, expert, and binary opinion, capturing expert-annotated structural views collectively.

Definition 2.7 (Soft Expert Graph) *Let $G = (V, E)$ be a (finite) simple graph as above, and let P be a nonempty set of parameters. Let X be a nonempty set of experts, and let*

$$O := \{1, 0\}$$

be the set of opinions, where 1 stands for "agree" and 0 for "disagree". Set

$$\Lambda := P \times X \times O$$

and let $A \subseteq \Lambda$ be a nonempty index set.

A mapping

$$F : A \longrightarrow \mathcal{P}(V)$$

is called a soft expert vertex assignment on G. For each $\alpha = (p, x, o) \in A$ we define

$$V_\alpha := F(\alpha) \subseteq V,$$

$$E_\alpha := \Big\{ \{u, v\} \in E : u, v \in V_\alpha \Big\},$$

and write

$$G_\alpha := (V_\alpha, E_\alpha).$$

The triple

$$SEG = \big(G, (F, A)\big)$$

is called a Soft Expert Graph over G if, for every $\alpha \in A$, the graph $G_\alpha = (V_\alpha, E_\alpha)$ is a vertex-induced subgraph of G.

Thus a Soft Expert Graph is a parameterized family $\{G_\alpha : \alpha \in A\}$ of induced subgraphs of G, where each index $\alpha = (p, x, o)$ jointly records a parameter $p \in P$, an expert $x \in X$, and that expert's opinion $o \in O$.

Example 2.3 (Soft Expert Graph) *Let $G = (V, E)$ be the simple undirected graph with*

$$V = \{v_1, v_2, v_3, v_4\},$$

$$E = \Big\{ \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_1\}, \{v_3, v_4\} \Big\},$$

so v_1, v_2, v_3 form a triangle and v_4 is attached to v_3.

Take the parameter set

$$P = \{p_{dense}, p_{peripheral}\},$$

where p_{dense} means "densely connected region" and $p_{peripheral}$ means "peripheral area of the network". Let

$$X = \{x_1, x_2\}, \qquad O = \{0, 1\},$$

with 1 = "agree" and 0 = "disagree". Set

$$\Lambda = P \times X \times O$$

and choose the nonempty index set

$$A = \{(p_{dense}, x_1, 1), (p_{dense}, x_2, 1), (p_{peripheral}, x_1, 1)\} \subseteq \Lambda.$$

Define

$$F : A \longrightarrow \mathcal{P}(V)$$

by

$$F(p_{dense}, x_1, 1) = \{v_1, v_2, v_3\},$$

$$F(p_{dense}, x_2, 1) = \{v_2, v_3\},$$

$$F(p_{peripheral}, x_1, 1) = \{v_3, v_4\}.$$

For each $\alpha \in A$ we then obtain

$$V_\alpha := F(\alpha), \qquad E_\alpha := \Big\{ \{u, v\} \in E : u, v \in V_\alpha \Big\},$$

and the induced subgraphs

$$G_\alpha := (V_\alpha, E_\alpha).$$

Explicitly,

$$G_{(p_{dense}, x_1, 1)} = \big(\{v_1, v_2, v_3\}, \{\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_1\}\}\big),$$

$$G_{(p_{dense}, x_2, 1)} = \big(\{v_2, v_3\}, \{\{v_2, v_3\}\}\big),$$

$$G_{(p_{peripheral}, x_1, 1)} = \big(\{v_3, v_4\}, \{\{v_3, v_4\}\}\big).$$

Thus

$$SEG = \big(G, (F, A)\big)$$

is a Soft Expert Graph: it is a family of induced subgraphs of G, each indexed by a triple $(p, x, o) \in A$ encoding a parameter, an expert, and the expert's opinion.

3. MAIN RESULT

This section presents the principal findings and theoretical contributions established in this paper.

3.1 MultiSoft Graph

We now introduce a multilevel parameter version of a soft graph, called a MultiSoft Graph. Its parameter structure mirrors the multisoft set in the Definition, while its vertex assignment produces induced subgraphs of a base graph.

Definition 3.1 (MultiSoft Graph). Let $G = (V, E)$ be a finite simple graph. Let $\{P_i\}_{i=1}^n$ be pairwise disjoint, nonempty parameter sets, and put

$$P := \bigcup_{i=1}^{n} P_i.$$

Let $A \subseteq \mathcal{P}(P)$ be a nonempty family of parameter subsets. A mapping

$$F : A \longrightarrow \mathcal{P}(V)$$

is called a MultiSoft vertex assignment on G.

For each $a \in A$, set

$$V_a := F(a) \subseteq V,$$

$$E_a := \left\{ \{u, v\} \in E \mid u, v \in V_a \right\},$$

and define the induced subgraph

$$G_a := (V_a, E_a).$$

The quadruple

$$\mathrm{MSG} = \left(G, (F, A), \{P_i\}_{i=1}^{n} \right)$$

is called a MultiSoft Graph over G if, for every $a \in A$, the graph $G_a = (V_a, E_a)$ is a vertex-induced subgraph of G. Equivalently, a MultiSoft Graph is a family $\{G_a : a \in A\}$ of induced subgraphs indexed by a multilevel parameter structure $\{P_i\}_{i=1}^{n}$ and their subsets $a \subseteq P$.

Example 3.2 (MultiSoft Graph: store selection under multi-criteria). Let $G = (V, E)$ be a simple graph of four retail stores,

$$V = \{s_1, s_2, s_3, s_4\},$$

where an edge indicates that the two stores are in the same shopping area:

$$E = \left\{ \{s_1, s_2\}, \{s_2, s_3\}, \{s_3, s_4\} \right\}.$$

Consider two parameter blocks

$$P_1 = \{p_{\mathrm{low}}, p_{\mathrm{medium}}\},$$

$$P_2 = \{q_{\mathrm{electronics}}, q_{\mathrm{groceries}}\},$$

where P_1 describes price level and P_2 describes the main product category. Put

$$P := P_1 \cup P_2.$$

Define a nonempty index family

$$A = \{a_1, a_2\} \subseteq \mathcal{P}(P)$$

by

$$a_1 = \{p_{\mathrm{low}}, q_{\mathrm{groceries}}\},$$

$$a_2 = \{p_{\mathrm{medium}}, q_{\mathrm{electronics}}\}.$$

Let

$$F : A \longrightarrow \mathcal{P}(V)$$

be given by

$$F(a_1) = \{s_1, s_2\},$$

$$F(a_2) = \{s_2, s_3, s_4\}.$$

For each $a \in A$ define

$$V_a := F(a),$$

$$E_a := \left\{ \{u, v\} \in E : u, v \in V_a \right\},$$

$$G_a := (V_a, E_a).$$

Then

$$\mathrm{MSG} = \left(G, (F, A), \{P_1, P_2\} \right)$$

is a MultiSoft Graph: for a_1 we obtain the subgraph induced by $\{s_1, s_2\}$ (low-price grocery stores in the same area), and for a_2 the subgraph induced by $\{s_2, s_3, s_4\}$ (medium-price electronics corridor).

We next prove that a MultiSoft Graph generalizes a Soft Graph and that its vertex assignment carries a natural MultiSoft Set structure on the vertex set.

Theorem 3.3 (Soft Graph as a special case; MultiSoft Set structure). Let

$$\mathrm{MSG} = \left(G, (F, A), \{P_i\}_{i=1}^{n} \right)$$

be a MultiSoft Graph over a finite simple graph $G = (V, E)$.

(1) **(Soft Graph as a special case)** Assume $n = 1$ and write $P_1 = P$. Suppose that A is of the form

$$A = \left\{ \{p\} \mid p \in A_0 \right\}$$

for some nonempty subset $A_0 \subseteq P$. Define

$$F^{\flat} : A_0 \longrightarrow \mathcal{P}(V),$$

$$F^{\flat}(p) := F(\{p\}) \quad (p \in A_0).$$

Then

$$\mathrm{SG} := \left(G, (F^{\flat}, A_0) \right)$$

is a Soft Graph over G in the sense of the Definition. Hence MultiSoft Graphs generalize Soft Graphs.

(2) **(Underlying MultiSoft Set)** Let $U := V$ and set

$$E_i := P_i \quad (1 \leq i \leq n),$$

$$E := \bigcup_{i=1}^{n} E_i = P.$$

Then the pair (F, A) is a multisoft set over U in the sense of the Definition. In particular, every MultiSoft Graph induces canonically a MultiSoft Set structure on its vertex set.

Proof.

(i) Assume $n = 1$ so that $\{P_1\}$ is a single parameter block and $P = P_1$. By assumption,

$$A = \{\{p\} \mid p \in A_0\}$$

for a nonempty subset $A_0 \subseteq P$. Define

$$\not{F} : A_0 \longrightarrow \mathcal{P}(V),$$
$$\not{F}(p) := F(\{p\}).$$

For each $p \in A_0$ we have, by Definition 3.1,

$$V_{\{p\}} = F(\{p\}) = \not{F}(p) \subseteq V,$$

and

$$E_{\{p\}} = \{\{u, v\} \in E \mid u, v \in V_{\{p\}}\} = \{\{u, v\} \in E \mid u, v \in \not{F}(p)\}.$$

The MultiSoft Graph condition states that

$$G_{\{p\}} := (V_{\{p\}}, E_{\{p\}})$$

is a vertex-induced subgraph of G for every $p \in A_0$.

Now, if we denote

$$V_p := \not{F}(p),$$
$$E_p := \{\{u, v\} \in E \mid u, v \in V_p\},$$
$$G_p := (V_p, E_p),$$

then for each $p \in A_0$ the graph G_p is vertex-induced in G. Thus the triple

$$\mathrm{SG} := (G, (\not{F}, A_0))$$

satisfies exactly the axioms of a Soft Graph. Therefore any MultiSoft Graph whose parameter subsets in A are singletons yields a Soft Graph via the identification

$$p \longleftrightarrow \{p\},$$
$$\not{F}(p) = F(\{p\}).$$

In particular, Soft Graphs embed as a special class of MultiSoft Graphs, so MultiSoft Graphs strictly generalize Soft Graphs.

(ii) By Definition 3.1, we have a family of pairwise disjoint parameter sets $\{P_i\}_{i=1}^n$ and their union

$$P = \bigcup_{i=1}^{n} P_i.$$

Set

$$U := V,$$
$$E_i := P_i \quad (1 \le i \le n),$$
$$E := \bigcup_{i=1}^{n} E_i = P.$$

We also have a nonempty index family $A \subseteq \mathcal{P}(P) = \mathcal{P}(E)$ and a mapping

$$F : A \longrightarrow \mathcal{P}(V) = \mathcal{P}(U).$$

Comparing with the Definition, we see that:

- U is a nonempty universe (the vertex set V of G).
- $\{E_i\}_{i=1}^n$ is a family of pairwise disjoint parameter sets, with union E.
- $A \subseteq \mathcal{P}(E)$ is nonempty.
- $F : A \to \mathcal{P}(U)$ assigns to each $a \in A$ an approximate subset $F(a) \subseteq U$.

Therefore the pair (F, A) satisfies all the requirements of a multisoft set over U. Concretely, for each $a \in A$ the value $F(a)$ is exactly the approximate value prescribed for the parameter subset $a \subseteq E$.

Hence every MultiSoft Graph

$$\mathrm{MSG} := (G, (F, A), \{P_i\}_{i=1}^n)$$

canonically induces a multisoft set (F, A) on the universe $U = V$. This proves (ii) and completes the proof.

3.2 Soft MultiExpert Graph

We next combine the Soft MultiExpert Set structure with a base graph. A Soft MultiExpert Graph assigns each parameter a multiset of expert-opinion induced subgraphs of a base graph, capturing repeated views.

Definition 3.4 (Soft MultiExpert Graph). Let $G = (V, E)$ be a finite simple graph, P a nonempty set of parameters, X a nonempty set of experts, and $O = \{0, 1\}$ a set of binary opinions. A Soft MultiExpert Graph over G is a pair

$$(G, (F, P))$$

where

$$F : P \longrightarrow M_{X \times O \times \mathcal{P}(V)}$$

is a mapping such that, for each parameter $p \in P$ and each triple

$$(x, o, V_{p,x,o}) \quad (x \in X,\ o \in O,\ V_{p,x,o} \subseteq V)$$

in the support of the multiset $F(p)$, the subgraph

$$G_{p,x,o} := (V_{p,x,o}, E_{p,x,o})$$

with

$$E_{p,x,o} := \{\{u, v\} \in E \mid u, v \in V_{p,x,o}\}$$

is a vertex-induced subgraph of G.

Intuitively, for each parameter p, the multiset $F(p)$ collects multiple expert–opinion views

$$(x, o, V_{p,x,o})$$

of the graph, each such view specifying an induced subgraph $G_{p,x,o}$ of G.

Remark 3.5. If, for every $p \in P$, the multiset $F(p)$ contains at most one copy of each triple $(x, o, V_{p,x,o})$, then the Soft MultiExpert Graph reduces to a Soft Expert Graph after forgetting the multiplicities.

Example 3.6 (Soft MultiExpert Graph: assessing a hospital network). Let $G = (V, E)$ be a simple graph describing three hospitals,

$$V = \{h_1, h_2, h_3\},$$

with edges representing direct patient-transfer agreements:

$$E = \Big\{ \{h_1, h_2\}, \{h_2, h_3\} \Big\}.$$

Take a parameter set

$$P = \{p_{\text{covid}}, p_{\text{cardio}}\},$$

where p_{covid} stands for "COVID-19 treatment capacity" and p_{cardio} for "cardiology specialization". Let the expert set and opinions be

$$X = \{x_{\text{gov}}, x_{\text{ins}}\}, \qquad O = \{1, 0\},$$

where x_{gov} is a public-health authority, x_{ins} an insurance analyst, and 1 (resp. 0) means "approve" (resp. "reject").

Define

$$F : P \longrightarrow M_{X \times O \times \mathcal{P}(V)}$$

by specifying the nonzero multiplicities.

For p_{covid} put

$$F(p_{\text{covid}})\big(x_{\text{gov}}, 1, \{h_1, h_2\}\big) = 2,$$
$$F(p_{\text{covid}})\big(x_{\text{ins}}, 1, \{h_2\}\big) = 1,$$

and $F(p_{\text{covid}})(y) = 0$ for all other y. Thus x_{gov} strongly supports the corridor $\{h_1, h_2\}$ for COVID-19 cases, while x_{ins} mildly supports h_2 alone.

For p_{cardio} set

$$F(p_{\text{cardio}})\big(x_{\text{gov}}, 1, \{h_2, h_3\}\big) = 1,$$
$$F(p_{\text{cardio}})\big(x_{\text{ins}}, 1, \{h_3\}\big) = 2,$$

and $F(p_{\text{cardio}})(y) = 0$ otherwise.

For each triple $(x, o, V_{p,x,o})$ in the support of $F(p)$ we obtain the induced subgraph

$$G_{p,x,o} := \Big(V_{p,x,o}, \{\{u, v\} \in E : u, v \in V_{p,x,o}\}\Big),$$

e.g. for p_{covid} and $\big(x_{\text{gov}}, 1, \{h_1, h_2\}\big)$ we get the subgraph induced by $\{h_1, h_2\}$. Consequently,

$$\big(G, (F, P)\big)$$

is a Soft MultiExpert Graph: each parameter collects a finite multiset of expert–opinion views, each view being an induced subgraph of the hospital network.

3.3 MultiSoft MultiExpert Graph

Finally, we introduce a MultiSoft version of the Soft Expert Graph, whose index sets incorporate both multilevel parameters and expert–opinion tuples, in parallel with the MultiSoft MultiExpert Set.

Definition 3.7 (MultiSoft MultiExpert Graph). Let $G = (V, E)$ be a finite simple graph. Let $\{P_i\}_{i=1}^n$ be pairwise disjoint, nonempty parameter sets with union

$$\bigcup_{i=1}^{n} P_i =: P.$$

Let X be a nonempty set of experts and $O = \{0, 1\}$ a set of binary opinions. Put

$$Z := P \times X \times O.$$

Let

$$A \in M_{\mathcal{P}(Z)}$$

be a finite multiset of subsets of Z. A mapping

$$F : \mathcal{P}(Z) \longrightarrow \mathcal{P}(V)$$

is said to define a MultiSoft MultiExpert Graph over G if, for every $\alpha \subseteq Z$ with $A(\alpha) > 0$, we have a vertex set

$$V_\alpha := F(\alpha) \subseteq V$$

and the corresponding induced subgraph

$$G_\alpha := (V_\alpha, E_\alpha), \qquad E_\alpha := \{\{u, v\} \in E \mid u, v \in V_\alpha\},$$

is a vertex-induced subgraph of G.

The data

$$\text{MSMEG} := \big(G, (F, A), \{P_i\}_{i=1}^n, X, O\big)$$

is then called a MultiSoft MultiExpert Graph over G.

Remark 3.8. If $n = 1$ and A is concentrated on subsets of the form

$$\alpha = \{(p, x, o)\} \quad (p \in P,\ x \in X,\ o \in O),$$

then a MultiSoft MultiExpert Graph reduces to a Soft Expert Graph. If in addition X and O are singletons, the structure further collapses to a MultiSoft Graph. Thus MultiSoft MultiExpert Graphs unify both MultiSoft Graphs and Soft Expert Graphs in a single framework.

Example 3.9 (Supplier portfolio design). Consider a company that must choose a portfolio of suppliers and logistic hubs. Let the underlying simple graph $G = (V, E)$ be given by

$$V = \{v_1, v_2, v_3, v_4\},$$

where v_1, v_2, v_3 denote candidate suppliers and v_4 denotes a central distribution hub. The edge set is

$$E = \Big\{ \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_1, v_2\} \Big\},$$

so that each supplier is connected to the hub and v_1, v_2 are mutually compatible.

We introduce three parameter blocks:

$$P_1 = \{p_{\text{cost low}}, p_{\text{cost medium}}\}, \quad P_2 = \{p_{\text{reliab high}}, p_{\text{reliab medium}}\}$$

$$P_3 = \{p_{\text{sust high}}\},$$

representing cost tier, reliability tier, and sustainability emphasis, respectively. Put

$$P := P_1 \cup P_2 \cup P_3.$$

Let the expert set and opinions be

$$X = \{x_{\text{proc}}, x_{\text{eng}}, x_{\text{sust}}\}, \quad O = \{1, 0\},$$

where x_{proc} is the procurement manager, x_{eng} the engineering lead, x_{sust} the sustainability officer, and 1 (resp. 0) stands for "approve" (resp. "reject"). Define

$$Z := P \times X \times O.$$

We now specify two subsets of Z that capture typical multi-parameter and multi-expert scenarios:

$$\alpha_1 := \big\{ (p_{\text{cost low}}, x_{\text{proc}}, 1), (p_{\text{reliab high}}, x_{\text{eng}}, 1), (p_{\text{sust high}}, x_{\text{sust}}, 1) \big\},$$

$$\alpha_2 := \big\{ (p_{\text{cost medium}}, x_{\text{proc}}, 1), (p_{\text{reliab medium}}, x_{\text{eng}}, 1) \big\}.$$

Define a finite multiset

$$A \in M_{\mathcal{P}(Z)}$$

by

$$A(\alpha_1) = 2, \qquad A(\alpha_2) = 1,$$

and $A(\beta) = 0$ for every other $\beta \subseteq Z$. Thus the scenario α_1 (strong joint emphasis on low cost, high reliability, and sustainability) is counted with multiplicity 2 in the overall assessment, while α_2 (medium cost and medium reliability) appears once.

Define

$$F : \mathcal{P}(Z) \longrightarrow \mathcal{P}(V)$$

by

$$F(\alpha_1) = \{v_1, v_2, v_4\}, \qquad F(\alpha_2) = \{v_2, v_3, v_4\},$$

and $F(\beta) = \varnothing$ whenever $A(\beta) = 0$. For each $\alpha \subseteq Z$ with $A(\alpha) > 0$ we put

$$V_\alpha := F(\alpha),$$

$$E_\alpha := \big\{ \{u, v\} \in E : u, v \in V_\alpha \big\},$$

$$G_\alpha := (V_\alpha, E_\alpha).$$

Explicitly,

$$G_{\alpha_1} = \big(\{v_1, v_2, v_4\}, \{\{v_1, v_4\}, \{v_2, v_4\}, \{v_1, v_2\}\}\big),$$

$$G_{\alpha_2} = \big(\{v_2, v_3, v_4\}, \{\{v_2, v_4\}, \{v_3, v_4\}\}\big).$$

By construction, each G_α is a vertex-induced subgraph of G for every α with $A(\alpha) > 0$. Hence the data

$$\text{MSMEG} := \big(G, (F, A), \{P_i\}_{i=1}^{3}, X, O\big)$$

form a MultiSoft MultiExpert Graph. It compactly encodes how several experts, under different multi-parameter configurations, collectively recommend particular supplier–hub subnetworks.

4. CONCLUSIONS

In this note, we introduced MultiSoft Graphs as multilevel, parameterized families of induced subgraphs of a base graph, and we established that they (i) extend the classical notion of Soft Graphs and (ii) naturally induce a canonical MultiSoft Set on the underlying vertex set. We also formulated Soft MultiExpert Graphs and MultiSoft MultiExpert Graphs as graph-theoretic analogues of Soft MultiExpert Sets and MultiSoft MultiExpert Sets, thereby providing a unified framework in which graph structure, multiple parameter blocks, and expert evaluations are treated jointly.

Future work includes examining further extensions based on Fuzzy Graphs [37, 38], Neutrosophic Graphs [39, 40, 41], Quadri-Partitioned Neutrosophic Graphs [42], HyperGraphs [23, 43], Neutrosophic HyperGraphs [44, 45], SuperHyperGraphs [25, 26], Rough Graphs [46], and Plithogenic Graphs [47, 48].

5. ACKNOWLEDGEMENT

We extend our sincere gratitude to everyone who provided insights, inspiration, and assistance throughout this research. We particularly thank our readers for their interest and acknowledge the authors of the cited works for laying the foundation that made our study possible. We also appreciate the support from individuals and institutions that provided the resources and infrastructure needed to produce and share this paper. Finally, we are grateful to all those who supported us in various ways during this project.

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