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Research Paper

On Square-Closed Lie Ideals and Generalized Homoderivations in Prime Rings

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Abstract
Let M be a square-closed noncentral Lie ideal of a prime ring R with char $R \neq 2$. An additive mapping G on R is defined as a *generalized homoderivation* if it satisfies $G(\sigma\tau) = G(\sigma)h(\tau) + G'(\sigma)y + xh(\tau)$ for all $\sigma, \tau \in R$. This paper focuses on studying generalized homoderivations of prime rings using square-closed Lie ideals that satisfy certain differential identities.

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1. INTRODUCTION

Throughout this paper, R consistently denote an associative ring with Z at its centre. The notion $[r_1, r_2]$ represent for the commutator $r_1r_2 - r_2r_1$ and the notion $r_1 \circ r_2$ represent for the anti-commutator $r_1r_2 + r_2r_1$ for any $r_1, r_2 \in R$. A ring R is considered semiprime if and only if $r_1Rr_1 = \{0\}$ symbolizes $r_1 = 0$. For any $r_1 \in M, r \in R$, a Lie ideal M is an additive subgroup of R if $[r_1, r] \in M$, and M is defined as a square-closed Lie ideal of R if M exists as a Lie ideal and $r_1^2 \in M$ for every $r_1 \in M$. If $r_1^2 \in M$ for every $r_1 \in M$, then $(r_1 + r_2)^2 \in M$ and accordingly $(r_1 + r_2)^2 - r_1^2 - r_2^2 = r_1r_2 + r_2r_1 \in M$, for all $r_1, r_2 \in M$. Also $r_1r_2 - r_2r_1 \in M$ for all $r_1, r_2 \in M$. Hence, we have $2r_1r_2 \in M$, for every $r_1, r_2 \in M$.

An additive function $d : R \to R$ is referred to as a derivation if $d(\sigma\tau) = d(\sigma)\tau + \sigma d(\tau)$ holds for all $\sigma, \tau \in R$. Over the past few decades, many researchers have explored the connection between the commutativity of a ring R and various specific types of derivations on R [1, 2, 3, 4, 5, 6, 7, 8, 9].

A homoderivation h on R is an additive function such that $h(\sigma\tau) = h(\sigma)h(\tau) + h(\sigma)\tau + \sigma h(\tau)$, for all $\sigma, \tau \in R$ in [10]. An illustration of this kind of function is $h(\sigma) = f(\sigma) - \sigma$, for all $\sigma \in R$ where f acts as an endomorphism of R. Clearly, a homoderivation h is also a derivation if $h(\sigma)h(\tau) = 0$ for every $\sigma, \tau \in R$.

An additive map $G : R \to R$ is defined as a right generalized homoderivation where there exist a homoderivation $h : R \to R$ for which $G(\sigma\tau) = G(\sigma)h(\tau) + G(\sigma)\tau + \sigma h(\tau)$ for all $\sigma, \tau \in R$ and G is referred to as a left generalized homoderivation wherein exists a homoderivation $h : R \to R$ provided that $G(\sigma\tau) = h(\sigma)G(\tau) + h(\sigma)\tau + \sigma G(\tau)$ for all $\sigma, \tau \in R$.

Let denote a generalized homoderivation, having an associated homoderivation. It is both right as well as left generalized homoderivation with an associated homoderivation. If $S \subset R$, then a function f on R holds S if $f(S) \subset S$. If f holds S and a function $f : R \to R$ is zero power valued on S for every $x \in S$, then the positive integer $n(x) > 1$ exists where $f^{n(x)}(x) = 0$ [10].

In [11], Daif and Bell studied that M is a nonzero ideal and d is a derivation of a semiprime ring R such that $d([\sigma, \tau]) = \pm[\sigma, \tau]$, for all $\sigma, \tau \in R$, then $M \subset Z$. In [12], Rehman et al. have studied the aforementioned statements related to homoderivations. In 2018, E.F. Alharfie and Muthana [13] proved the commutativity in prime rings with homoderivations. Recently in 2023, Boua [14] develops results related to generalized homoderivation and also examines semiprime rings that admit such generalized homoderivations shows commutativity.

Several researchers have investigated the relationship between prime and semiprime rings and the action of homoderivations on suitable subsets of R [15, 16, 17, 18, 19, 20], where further references can be found. In this line of investigation, we characterize some results on generalized homoderivations in a prime ring using square-closed Lie ideals which satisfy certain differential identities.

2. METHODS

Throughout this paper, M is a noncentral square-closed Lie ideal and R a prime ring whose char $R \neq 2$. For any $\sigma, \tau, \varpi \in R$, we extensively use basic commutator identities:

$$
\begin{aligned}
[\sigma, \tau\varpi] &= \tau[\sigma, \varpi] + [\sigma, \tau]\varpi, \\
[\sigma\tau, \varpi] &= [\sigma, \varpi]\tau + \sigma[\tau, \varpi], \\
\sigma \circ \tau\varpi &= (\sigma \circ \tau)\varpi - \tau[\sigma, \varpi] = \tau(\sigma \circ \varpi) + [\sigma, \tau]\varpi, \\
(\sigma\tau) \circ \varpi &= \sigma(\tau \circ \varpi) - [\sigma, \varpi]\tau = (\sigma \circ \varpi)\tau + \sigma[\tau, \varpi].
\end{aligned}
$$

We begin with the following known results, which will be used to prove our theorems.

Lemma 2.1. ([12, Lemma 4]) Let M be a noncentral Lie ideal of a prime ring R whose char $R \neq 2$. If $\sigma M \tau = (0)$ then $\sigma = 0$ or $\tau = 0$.

Lemma 2.2. Let M be a noncentral Lie ideal of a prime ring R whose char $R \neq 2$ and $\sigma \in R$. If $M\sigma = (0)$ ($\sigma M = (0)$), then $\sigma = 0$.

Proof.

Consider an arbitrary nonzero τ in R. $M\sigma = (0)$ infers $\tau M \sigma = (0)$. So, then $\sigma = 0$ immediately follows from Lemma 2.1.

Lemma 2.3. ([15, Theorem 2]) Let M be a noncentral Lie ideal in a prime ring R whose char $R \neq 2$ and h a homoderivation in R. If $h(M) \subseteq Z$, then $h = 0$.

Lemma 2.4. ([15, Lemma 9]) Let M be a noncentral Lie ideal of a prime ring R whose char $R \neq 2$, $\sigma \in R$. If $\sigma \circ M = (0)$, then $\sigma \in Z$.

3. RESULTS AND DISCUSSION

Theorem 3.1. Let M be a noncentral square-closed Lie ideal and R a prime ring whose char $R \neq 2$. Assume that R is associated with a generalized homoderivation G corresponding to a homoderivation h, where h maps M to zero power values. When any of the following statements hold then $h = 0$.

1. $h(\sigma)G(\tau) = [\sigma, \tau]$, for all $\sigma, \tau \in M$.
2. $h(\sigma)G(\tau) = \sigma \circ \tau$, for all $\sigma, \tau \in M$.

Proof.

(i) By assumption, we have

$$h(\sigma)G(\tau) = [\sigma, \tau] \quad \text{for all } \sigma, \tau \in M. \tag{3.1}$$

By replacing τ with $2\tau\sigma$ in (3.1), we obtain

$$
\begin{aligned}
&2h(\sigma)G(\tau)h(\sigma) + 2h(\sigma)G(\tau)\sigma + 2h(\sigma)\tau h(\sigma) \\
&= 2[\sigma, \tau]\sigma \quad \text{for all } \sigma, \tau \in M.
\end{aligned} \tag{3.2}
$$

Use (3.1) in (3.2) and also apply char $R \neq 2$, it follows that

$$h(\sigma)G(\tau)h(\sigma) + h(\sigma)\tau h(\sigma) = 0 \quad \text{for all } \sigma, \tau \in M. \tag{3.3}$$

That is,

$$h(\sigma)(G(\tau) + \tau)h(\sigma) = 0 \quad \text{for all } \sigma, \tau \in M.$$

Since G takes zero power valued on M, an integer $n = n(\tau) > 1$ is said to exist such that $G^n(\tau) = 0$ holds. Replacing τ by $\tau - G(\tau) + G^2(\tau) + \cdots + (-1)^{n-1}G^{n-1}(\tau)$ in (3.3) we get

$$h(\sigma)\tau h(\sigma) = 0 \quad \text{for all } \sigma, \tau \in M.$$

That is, $h(\sigma)Mh(\sigma) = (0)$, for all $\sigma \in M$. By Lemma 2.1, we have $h(\sigma) = (0)$ for every $\sigma \in M$, from this it is concluded that $h = 0$ according to Lemma 2.3.

(ii) Let it be assumed that

$$h(\sigma)G(\tau) = \sigma \circ \tau \quad \text{for all } \sigma, \tau \in M. \tag{3.4}$$

By replacing τ with $2\tau\sigma$ in (3.4), we obtain

$$
\begin{aligned}
&2h(\sigma)G(\tau)h(\sigma) + 2h(\sigma)G(\tau)\sigma + 2h(\sigma)\tau h(\sigma) \\
&= 2(\sigma \circ \tau)\sigma \quad \text{for all } \sigma, \tau \in M.
\end{aligned} \tag{3.5}
$$

Using (3.4) in (3.5) and also use char $R \neq 2$, we have

$$h(\sigma)G(\tau)h(\sigma) + h(\sigma)\tau h(\sigma) = 0 \quad \text{for all } \sigma, \tau \in M. \tag{3.6}$$

The remaining proof follows identically to the argument in equation (3.3), thus completing the proof.

Theorem 3.2. Let M be a noncentral square-closed Lie ideal and R a prime ring whose char $R \neq 2$. Assume that R is associated with a generalized homoderivation G corresponding to a homoderivation h, where h maps M to zero power values. If $[G(\sigma), \tau] \pm \sigma\tau \in Z$, for all $\sigma, \tau \in M$, then $h = 0$.

Proof Let it be assume that

$$[G(\sigma), \tau] \pm \sigma\tau \in Z \text{ for all } \sigma, \tau \in M. \tag{3.7}$$

Substituting τ with $2\tau G(\sigma)$ in (3.7) and using hypothesis, we obtain

$$2([G(\sigma), \tau] \pm \sigma\tau)G(\sigma) \in Z, \text{ for all } \sigma, \tau \in M. \tag{3.8}$$

Since $[G(\sigma), \tau] \pm \sigma\tau \in Z$, char $R \neq 2$ and R is a prime ring, we have

$$G(\sigma) \in Z \quad \text{or} \quad [G(\sigma), \tau] \pm \sigma\tau = 0.$$

Let

$$
\begin{aligned}
K &= \{\sigma \in M : [G(\sigma), \tau] \pm \sigma\tau = 0, \text{ for every } \tau \in M\} \quad \text{and} \\
L &= \{\sigma \in M : G(\sigma) \in Z\}.
\end{aligned}
$$

The sets K and L are two additive subgroups of M and it holds that $M = L \cup K$. However, it is not possible for a group to be the union of two proper subgroups. Thus $K = M$ or $L = M$.

Suppose $K = M$. Then we have

$$[G(\sigma), \tau] \pm \sigma\tau = 0, \text{ for all } \sigma, \tau \in M. \tag{3.9}$$

Substituting τ by $2\sigma\tau$ in (3.9), we obtain that

$$2[G(\sigma), \sigma\tau] \pm 2\sigma^2\tau = 0.$$

That is,

$$2\sigma([G(\sigma), \tau] \pm \sigma\tau) + 2[G(\sigma), \sigma]\tau = 0,$$

and so

$$2[G(\sigma), \sigma]\tau = 0 \text{ for all } \sigma, \tau \in M.$$

Since char $R \neq 2$, we have

$$[G(\sigma), \sigma]\tau = 0 \text{ for all } \sigma, \tau \in M. \quad (3.10)$$

Lemma 1 results that

$$[G(\sigma), \sigma] = 0 \text{ for all } \sigma, \tau \in M. \quad (3.11)$$

In contrast, we get $[G(\sigma), \sigma] \pm \sigma^2 \in Z$ by (3.9). Using $[G(\sigma), \sigma] = 0$ for every $\sigma \in M$, we find $\sigma^2 \in Z$ for all $\sigma \in M$. Thus, we obtain

$$(\sigma + \tau)^2 = \sigma^2 + \tau^2 + \sigma\tau + \tau\sigma \in Z,$$

and so $\sigma\tau + \tau\sigma \in Z$ for all $\sigma, \tau \in M$. Replacing σ by $2\sigma\tau$ in this equation and using the same, we get $2(\sigma\tau + \tau\sigma)\tau \in Z$ for all $\sigma \in M$. Thus, $\sigma\tau + \tau\sigma \in Z$, char $R \neq 2$ also considering the primeness of R, it follows that $\tau \in Z$ or $\sigma\tau + \tau\sigma = 0$. Define

$$L = \{\tau \in M : \tau \in Z\} \quad \text{and} \quad K = \{\tau \in M : \sigma\tau + \tau\sigma = 0, \text{ for every } \tau \in M\}.$$

Apply Brauer's Trick, which forces that $K = M$ or $L = M$. In the initial case, $M \subseteq Z$ which compels that the hypothesis. Consequently, we need to $\sigma\tau + \tau\sigma = 0$ for every $\sigma \in M$. Thus, we conclude that $M \circ M = 0$, and thus $M \subseteq Z$ according to Lemma 2.4. Which contradicts that $M \nsubseteq Z$. From this, it follows that $L = M$, and so $G(\sigma) \in Z$ for every $\sigma \in M$. We find that $G = 0$ by Lemma 2.3. Thus, the proof is concluded.

Theorem 3.3 Let M be a noncentral square-closed Lie ideal and R a prime ring whose char $R \neq 2$. Assume that R is associated with a generalized homoderivation G corresponding to a homoderivation h, where h maps M to zero power values. If $\tau G(\sigma) \pm \sigma\tau \in Z$, for all $\sigma, \tau \in M$, then $h = 0$.
Proof. Let it be assume that

$$\tau G(\sigma) \pm \sigma\tau \in Z \text{ for all } \sigma, \tau \in M. \quad (3.12)$$

Replacing τ by $2\sigma\tau$ in (3.12), we have

$$2\sigma(\tau G(\sigma) + \sigma\tau) \in Z \text{ for all } \sigma, \tau \in M. \quad (3.13)$$

Considering that $\sigma(\tau G(\sigma) + \sigma\tau) \in Z$ and a prime ring R with char $R \neq 2$, it follows that $\sigma \in Z$ or $\tau G(\sigma) + \sigma\tau = 0$. Now apply Brauer's Trick, then we have $\tau G(\sigma) + \sigma\tau = 0$ for all $\sigma, \tau \in M$.

Writing σ by $2\sigma\tau$ in the above equation, it is obtained that

$$2\tau G(\sigma)h(\tau) + 2\tau G(\sigma)\tau + 2\tau\sigma h(\tau) + 2\tau\sigma\tau = 0 \text{ for all } \sigma, \tau \in M.$$

$$2\tau(G(\sigma) + \sigma)h(\tau) + 2(\tau G(\sigma) + \sigma\tau)\tau = 0 \text{ for all } \sigma, \tau \in M,$$

and so,

$$\tau(G(\sigma) + \sigma)h(\tau) = 0 \text{ for all } \sigma, \tau \in M. \quad (3.14)$$

Since G takes zero power valued on M, an integer $n = n(\tau) > 1$ is said to exist such that $G^n(\tau) = 0$ holds. Replacing τ by $\tau - G(\tau) + G^2(\tau) + \cdots + (-1)^{n-1}G^{(n-1)}(\tau)$ in the above equation, we get Lemma 2.1 yields that $\tau = 0$ or $h(\tau) = 0$ for each $\tau \in M$. If $\tau = 0$, then $h(\tau) = 0$. Hence, it follows that $h(\tau) = 0$ for all $\tau \in M$. The desired result follows from Lemma 2.3.

Theorem 3.4. Let M be a noncentral square-closed Lie ideal and R a prime ring whose char $R \neq 2$. Assume that R is associated with a generalized homoderivation G corresponding to a homoderivation h, where h maps M to zero power values. If $\tau G(\sigma) \pm [\sigma, \tau] \in Z$ for all $\sigma, \tau \in M$, then $h = 0$.
Proof. Let it be assume that

$$\tau G(\sigma) \pm [\sigma, \tau] \in Z \text{ for all } \sigma, \tau \in M. \quad (3.15)$$

Substituting τ with $2\sigma\tau$ in (3.15) and using (3.15), we arrive at

$$2\sigma(\tau G(\sigma) \pm [\sigma, \tau]) \in Z \text{ for all } \sigma, \tau \in M. \quad (3.16)$$

Considering that $\tau G(\sigma) \pm [\sigma, \tau] \in Z$ and a prime ring R whose char $R \neq 2$, this implies that $\sigma \in Z$ or $\tau G(\sigma) \pm [\sigma, \tau] \in Z$. Define

$$L = \{\sigma \in M : \sigma \in Z\} \quad \text{and} \quad K = \{\sigma \in M : \tau G(\sigma) \pm [\sigma, \tau] = 0 \text{ for every } \sigma, \tau \in M\}.$$

Upon applying Brauer's Trick, we reach at $K = M$ or $L = M$. In the initial case, $M \subseteq Z$, this enforces the hypothesis. As a result, we find that

$$\tau G(\sigma) \pm [\sigma, \tau] = 0 \text{ for every } \sigma, \tau \in M.$$

Substituting σ with $2\sigma\tau$ for the above equation, it is clear that

$$2\tau G(\sigma)h(\tau) + \tau G(\sigma)\tau + \tau\sigma h(\tau) \pm [\sigma, \tau]\tau = 0,$$

$$\tau(G(\sigma) + \sigma)h(\tau) + (\tau G(\sigma) \pm [\sigma, \tau])\tau = 0.$$

As $\tau G(\sigma) \pm [\sigma, \tau] = 0$ and also using char $R \neq 2$, we have

$$\tau(G(\sigma) + \sigma)h(\tau) = 0 \text{ for every } \sigma, \tau \in M.$$

Since G takes zero power valued on M, an integer $n = n(\tau) > 1$ is said to exist such that $G^n(\tau) = 0$ holds. Replacing τ by

$\tau - G(\tau) + G^2(\tau) + \cdots + (-1)^{n-1}G^{(n-1)}(\tau)$ in the above equation, we infer that

$$\tau\sigma h(\tau) = 0 \text{ for every } \sigma, \tau \in M,$$

Lemma 2.1 results that $\tau = 0$ or $h(\tau) = 0$ for each $\tau \in M$. If $\tau = 0$, then $h(\tau) = 0$. Hence, we have $h(\tau) = 0$ for every $\tau \in M$. The desired result follows from Lemma 2.3.

Theorem 3.5. Let M be a noncentral square-closed Lie ideal and R a prime ring whose char $R \neq 2$. Assume that R is associated with a generalized homoderivation G corresponding to a homoderivation h, where h maps M to zero power values. If $\tau G(\sigma) \pm (\sigma \circ \tau) \in Z$ for every $\sigma, \tau \in M$, then $h = 0$.
Proof. Assume that

$$\tau G(\sigma) \pm (\sigma \circ \tau) \in Z \text{ for all } \sigma, \tau \in M. \tag{3.17}$$

Substituting τ with $2\sigma\tau$ in (3.17) and using (3.17), we find that

$$2\sigma(\tau G(\sigma) \pm (\sigma \circ \tau)) \in Z \text{ for all } \sigma, \tau \in M. \tag{3.18}$$

Considering that $\tau G(\sigma) \pm (\sigma \circ \tau) \in Z$ and a prime ring R whose char $R \neq 2$, this implies that $\sigma \in Z$ or $\tau G(\sigma) \pm (\sigma \circ \tau) \in Z$. Define

$$L = \{\sigma \in M : \sigma \in Z\} \quad \text{and} \quad K = \{\sigma \in M : \tau G(\sigma) \pm (\sigma \circ \tau) = 0 \text{ for every } \sigma, \tau \in M\}. \tag{1}$$

Applying Brauer's Trick, we reach at $K = M$ or $L = M$. In the initial case, $M \subseteq Z$, this enforces the hypothesis. As a result, we find that

$$\tau G(\sigma) \pm (\sigma \circ \tau) = 0 \text{ for every } \sigma, \tau \in M.$$

Substituting σ with $2\sigma\tau$ for the above equation, it is clear that

$$2\tau G(\sigma)h(\tau) + \tau G(\sigma)\tau + \tau\sigma h(\tau) \pm (\sigma \circ \tau)\tau = 0,$$

$$\tau(G(\sigma) + \sigma)h(\tau) + (\tau G(\sigma) \pm (\sigma \circ \tau))\tau = 0.$$

As $\tau G(\sigma) \pm (\sigma \circ \tau) = 0$ and also using char $R \neq 2$, we have

$$\tau(G(\sigma) + \sigma)h(\tau) = 0 \text{ for every } \sigma, \tau \in M.$$

Since G takes zero power valued on M, an integer $n = n(\tau) > 1$ is said to exist such that $G^n(\tau) = 0$ holds. Replacing τ by $\tau - G(\tau) + G^2(\tau) + \cdots + (-1)^{n-1}G^{(n-1)}(\tau)$ in the above equation, we infer that

$$\tau\sigma h(\tau) = 0 \text{ for every } \sigma, \tau \in M,$$

Lemma 2.1 results that $\tau = 0$ or $h(\tau) = 0$ for each $\tau \in M$. If $\tau = 0$, then $h(\tau) = 0$. Hence, we have $h(\tau) = 0$ for every $\tau \in M$. The desired result follows from Lemma 2.3.

Example: Suppose

$$R = \left\{\begin{pmatrix} \alpha_1 & 0 & 0 \\ \alpha_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : \alpha_1, \alpha_2 \in R\right\}, \quad M = \left\{\begin{pmatrix} 0 & 0 & 0 \\ \beta_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : \beta_1 \in R\right\}.$$

M is a noncentral square-closed Lie ideal and R is a prime ring whose char $R \neq 2$. Let us define mappings $G, h : R \to M$ such that

$$G\left(\begin{pmatrix} \alpha_1 & 0 & 0 \\ \alpha_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad h\left(\begin{pmatrix} \alpha_1 & 0 & 0 \\ \alpha_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & 0 \\ \alpha_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It is obvious that there is a nonzero generalized homoderivation G having zero power value. Certain conditions that are commutative and enumerated above hold true. Although, R is non-commutative and M is noncentral.

4. CONCLUSIONS

Let R be a ring. An additive mapping $d : R \to R$ is termed a derivation if $d(\sigma\tau) = d(\sigma)\tau + \sigma d(\tau)$ for all $\sigma, \tau \in R$. The study of derivations in prime rings was first initiated by Posner [1]. Subsequently, numerous authors have investigated commutativity theorems for prime rings admitting automorphisms or derivations on suitable subsets of R. In 2000, El-Sofy [10] introduced the notion of a homoderivation on R, defined as an additive mapping $h : R \to R$ satisfying $h(\sigma\tau) = h(\sigma)h(\tau) + h(\sigma)\tau + \sigma h(\tau)$ for all $\sigma, \tau \in R$. Several significant results have been established concerning commutativity associated with homoderivations. The objective of this work is to establish results on generalized homoderivations in a prime ring through the study of square-closed Lie ideals that obey certain differential identities.

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