Integra: Journal of Integrated Mathematics and Computer Science
Journal Homepage: https://integrajimcs.com/index.php/integra/index

integra
Journal of Integrated Mathematics and Computer Science

Research Paper

Application of GSTARMA Spatial-Temporal Model for Inflation Analysis in South Sulawesi Province

Dede Ratna Sari[1], Widiarti[1*], Dina Eka Nurvazly[1], Mustofa Usman[1], Luvita Loves[2]

[1] *Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Lampung, Lampung, 35145, Indonesia*
[2] *Department of Mathematics, Faculty of Science and Technology, Universitas Pattimura, Ambon, 97233, Indonesia*
[*] **Corresponding author**: widiarti.1980@fmipa.unila.ac.id

Keywords
GSTARMA, Spatial-temporal Model, Inflation, South Sulawesi Province

Abstract
The Generalized Space-Time Autoregressive Moving Average (GSTARMA) model is a development of the time series model that can capture both spatial and temporal dynamics simultaneously. This study uses the GSTARMA model to analyze inflation data in five cities in South Sulawesi Province from January 2017 to October 2024. The GSTARMA model obtained is GSTARMA (1,0,1) with a cross-correlation normalization spatial weight matrix. The results of the analysis indicate a spatial influence between locations and temporal relationships in the inflation data.

Received: 23 September 2024, Accepted: 28 October 2024
https://doi.org/10.26554/integra.20241322

1. INTRODUCTION

A time series is a collection of observations organized sequentially according to time sequence [1]. Wei [2] defines two categories of time series: univariate and multivariate. In forecasting time series data, analysis is not only influenced by time factors but also by location factors. A space-time model is one that can combine both of these aspects.

The first space-time model introduced was Space-Time Autoregressive (STAR). The STAR model was then expanded into the STARMA model by adding a Moving Average component to account for random errors from previous times. The STARMA model defines that each location has the same characteristics, so it is not suitable if applied to locations that have different characteristics. The GSTARMA model can be used to analyze locations that have different characteristics (heterogeneous).

GSTARMA applications in economics are usually used to predict spatial (space) and temporal (time) phenomena that are interrelated such as inflation rates from several related regions based on historical data. Inflation in one city/district can influence inflation in other cities/districts, especially those that are geographically close or have close economic relations.

South Sulawesi Province inflation is calculated based on the combined consumer price index obtained from five cities, including Bulukumba City, Watampone City, Makassar City, Pare-pare City, and Palopo City. The diversity of South Sulawesi Province creates different inflation dynamics, making it suitable for analysis using the GSTARMA method which considers spatial relationships between regions.

Some previous research related to the integration of GSTARIMA models in various forecasting applications include the research by Munandar et al [3] which developed a hybrid model combining GSTARIMA with Deep Neural Networks (DNN) to enhance rainfall prediction accuracy. Salsabila et al [4] focused on developing the GSTARIMA (1,1,1) model order for climate data forecasting. They utilized the data analytics lifecycle method to analyse large-scale rainfall data, concluding that the GSTARIMA (3,1,1) model provided more accurate short-term forecasts, particularly for rainfall in West Java Province. Safira et al. [5] compared the performance of ARIMA and GSTARIMA models in predicting the spatial impact on inflation in Java Island Monika et al [6] conducted a systematic literature review on integrating the GSTARIMA model with heteroskedastic error and the Kriging method for climate forecasting. Aulia and Saputro [7] analysed the Generalized Space-Time Autoregressive Integrated Moving Average with Exogenous (GSTARIMA-X) models. They examined the model's capability to incorporate exogenous variables, enhancing the forecasting of space-time data by considering additional influencing factors. Akbar et al. [8] applied the GSTARMA model to forecast air pollutant levels in Surabaya. Ajobo et al [9] introduced the Generalized Space-Time Seasonal Autoregressive Integrated Moving Average Seemingly Unrelated Regression (GSTSARIMA-SUR) model to analyze seasonal and non-stationary data. Monika et al. [10] developed the GSTARI-X-ARCH model by integrating GSTARIMA with exogenous variables and ARCH models, using a data mining approach for forecasting climate in West Java, and Kurniawan et al [11] focused on modeling data following the GSTARMA-X model using Kalman filters. This approach aimed to enhance the estimation and forecasting of space-time data by leveraging the recursive capabilities of Kalman filtering techniques. These studies collectively contribute to the advancement of spatio-temporal modeling techniques, particularly in climate and environmental forecasting, by integrating GSTARIMA models with various statistical and machine learning methods.

Mukhaiyar et al. [12] applied a Generalized Space–Time Autoregressive Model incorporating a three-dimensional spatial weight matrix to predict water levels in Indonesian peatlands. Pasaribu et al. [13] used a similar autoregressive framework to analyse the vertical distribution of copper and gold grades, focusing on a porphyry-deposit case study. Mukhaiyar et al. [14] also introduced a Minimum Spanning Tree approach to construct the weight matrix within a Generalized Space–Time Autoregressive Model for modeling COVID-19 cases across Java Island. In another study, Mukhaiyar et al. [15] evaluated Space–Time Autoregressive Modeling with time-correlated errors to analyze vehicle counts passing through the Purbaleunyi toll gates. Huda & Imro'ah [16] investigated which spatial weight matrix is most effective for a Generalized Space–Time Model applied to COVID-19 case data on Java Island. Hestuningtias & Kurniawan [17] implemented the Generalized Space–Time Autoregressive Model to forecast inflation. Finally, Huda et al. [18] examined an approximation of the Generalized Space–Time Autoregressive model for discrete phenomena using the INAR (Integer-valued Autoregressive) framework.

Based on the background that has been presented, the author is interested in conducting an analysis using the GSTARIMA model with a differencing component ($d = 0$) for data that is already stationary or also called the GSTARMA model on inflation data in five cities in South Sulawesi Province to determine the predicted inflation value in five different cities that are influenced by the economic relationship between each city.

2. METHODS

The data used in this study are secondary, specifically inflation data from five cities in South Sulawesi province. Makassar City, Bulukumba City, Watampone City, Pare-pare City, and Palopo City are the five places used as research variables. The data period used is January 2017 to October 2024 with a total of 94 data. The selection of data from the previous 5 to 10 years will provide sufficient historical data to capture trends and dynamics of change between locations so that it is more representative of the period to be predicted. The use of data from only one year may not reflect the variability needed for the analysis. Inflation data from January 2017 to April 2023 is used as in-sample data and inflation data from May to October 2024 is used as out-sample data. This data selection is carried out based on the provisions of optimal data division with a ratio of 80:20, with 80% of the data used to build and estimate the model (in-sample), and the remaining 20% is used to test the ability of the forecast results (out-sample). Inflation data in the five cities were obtained from the Central Statistics Agency of South Sulawesi Province which can be accessed via https://sulsel.bps.go.id/id.

The steps used in analyzing inflation data using the GSTARMA method are as follows:

1. Conducting data exploration to determine data characteristics by identifying patterns and gaining an understanding of the data set.
2. Calculating the Gini index value to measure the level of heterogeneity of the research location. If the Gini index value is equal to 0, it means that there is no heterogeneity between regions, whereas if the Gini index value is equal to 1, then there is high heterogeneity between regions.
3. Conducting spatial autocorrelation testing. Spatial autocorrelation is the dependence of a particular variable value on the value of the same variable recorded at neighboring locations. The Moran index can be used as a tool to measure the level of spatial autocorrelation. That index is a global index to determine whether there is a spatial relationship in a particular event.
4. Conducting data stationarity checks. The research data is viewed for its stationarity against the variance and mean. If the data is not stationary against the variance, a Box-Cox transformation is used, and if the data is not stationary against the mean, differencing is applied.
5. Identification of GSTARMA model. Time order in the GSTARMA model is identified using AICC value with the criteria that the model order that has the smallest AICC value is considered the best model order. The GSTARMA model has a spatial order of one since greater orders are difficult to grasp.
6. Performing the formation and calculation of spatial weighting matrices using uniform location weights and cross-correlation normalized location weights.
7. Conducting GSTARMA model parameter estimation using the Ordinary Least Square (OLS) and Seemingly Unrelated Regression (SUR) approaches. The OLS method is used when the residuals between each equation are not correlated with each other, while the SUR method is used when the residuals between equations are related to each other or correlated with each other.
8. Model diagnostic test with a portmanteau test to determine whether the residuals are white noise and normally distributed multivariate by looking at the q-q plot.
9. Performing the selection of the best GSTARMA model with the smallest RMSE value.

10. Forecasting inflation data for several future periods in five cities using the best GSTARMA model.

3. RESULTS AND DISCUSSION

In this study, inflation data processing was carried out using SAS and Minitab software to obtain the best GSTARMA model and forecast inflation values for several periods into the future. The analysis was used to obtain a model that not only describes spatial and temporal relationships in inflation data but also provides predictions of inflation values for each location.

3.1 Descriptive Statistics

Descriptive statistics are used to describe and illustrate the information contained in the data. Table 1 shows the results of descriptive statistics on inflation values in five cities in South Sulawesi Province.

Table 1. Descriptive Inflation Data in Five Cities in South Sulawesi Province

Location	Average	Standard Deviation	Maximum Value	Minimum Value
Bulukumba	0.2666	0.3907	1.31	-0.60
Watampone	0.2961	0.5067	1.83	-0.69
Makassar	0.2846	0.4421	1.27	-0.85
Pare-pare	0.2779	0.6227	1.88	-1.59
Palopo	0.2738	0.4528	1.74	-0.69

Table 1 shows that the highest average inflation value of 0.2961 is in Watampone City, while the lowest average inflation value is in Bulukumba City with a value of 0.2666. The largest standard deviation is in Pare-pare City, which is 0.6227. This shows that inflation data in Pare-pare City is more spread out than the other four cities, while the lowest standard deviation is in Bulukumba City with a value of 0.3907, which shows that inflation data in Bulukumba City has a smaller level of data spread. The highest inflation value is in Pare-pare City with a value of 1.88, and the lowest inflation value is also in Pare-pare City with a value of -1.59.

3.2 Inflation Data Plot

Data plots are used as exploratory tools to understand the characteristics and patterns contained in the data. Figure 1 displays the combined data plot of inflation values from five cities.

Based on Figure 1, it can be seen that the five cities in South Sulawesi Province have relatively the same pattern in terms of exploration. This shows that there is a mutual influence effect between the five cities. The data plot also shows that inflation in the five cities in South Sulawesi Province has changed over time. Based on the data plot, it is known that inflation data fluctuates, but does not have a tendency to increase or decrease.

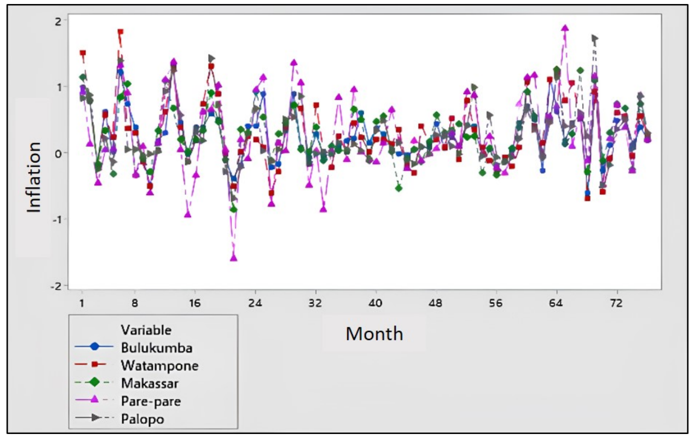

Figure 1. Plot of Inflation Value Data in Five Cities in South Sulawesi Province

3.3 Gini Index Inflation Data

Heterogeneity between locations can be measured using the Gini index, a higher Gini index value indicates a greater level of heterogeneity. The following is the calculation of the Gini index for inflation data in five cities:

$$
\begin{aligned}
IG &= 1 + \frac{1}{T} - \frac{2}{T^2 \bar{z}_t} \sum_{i=1}^{N} \sum_{t=1}^{T} z_t \\
&= 1 + \frac{1}{76} - \frac{2}{76^2 \cdot 0.2798} \cdot 106.32 \\
&= 0.8816
\end{aligned}
$$

Based on the calculation results above, the Gini index value obtained is 0.8816, this indicates heterogeneity between locations in inflation data.

Table 2. $p - Value$ Antar Lokasi

Location	Bulukumba	Watampone	Makassar	Pare-pare	Palopo
Bulukumba	1.0000	0.8235 < .0001	0.7571 < .0001	0.6506 < .0001	0.7086 < .0001
Watampone	0.8235 < .0001	1.0000	0.6729 < .0001	0.6867 < .0001	0.7721 < .0001
Makassar	0.7571 < .0001	0.6729 < .0001	1.0000	0.6693 < .0001	0.7034 < .0001
Pare-pare	0.6506 < .0001	0.6867 < .0001	0.6693 < .0001	1.0000	0.6704 < .0001
Palopo	0.7086 < .0001	0.7721 < .0001	0.7034 < .0001	0.6704 < .0001	1.0000

3.4 Correlation between Variable Locations

The correlation value is used to determine the direction of the relationship, the strength of the relationship, and the significance

of the strength of the relationship between each location variable used. The significance test of the correlation value can be done with the following hypothesis:
H_0 : $r_{ij} = 0$ (There is no correlation between location variables)
H_1 : $r_{ij} \neq 0$ (There is a correlation between location variables)

Table 2 displays the correlation values between location variables in data inflation. Based on the results of the correlation test in Table 2, it is known that each location variable has a $p - value < \alpha$ (0.05) which means that H_0 is rejected. So it can be concluded that there is a correlation between each city. The resulting correlation value shows a value greater than 0.5 so it can be said that there is a fairly strong correlation between each location variable.

Table 3. Results of Moran's Index Test of Inflation Data

I	E (I)	VAR (I)	Z(I)	$p - value$	Information
0.2744	-0.2500	0.0404	2.609	0.0045	Significant

3.5 Spatial Autocorrelation Testing

Spatial autocorrelation testing aims to identify spatial dependence on inflation values in five cities. Spatial autocorrelation testing can be done using the Moran index. According to Lee & Wong [19], the formula for the Moran index value can be written as follows.

$$I = \frac{N \sum_{i=1}^{N} \sum_{j=1}^{N} W_{ij}(Z_i - \bar{Z})(Z_j - \bar{Z})}{S_0 \sum_{i=1}^{N} (Z_i - \bar{Z})^2} \tag{1}$$

I : Moran index
N : Number of locations
Z_i : Observation value at location i
Z_j : Observation value at location j
$\bar{Z}$: Average number of observation values
W_{ij} : Location weighting element between areas i and j

Spatial autocorrelation can be tested with the following hypothesis:
H_0 : $I = 0$ (no spatial autocorrelation)
H_0 : $I \neq 0$ (there is spatial autocorrelation)

Table 3 shows the Results of Moran's Index Test of Inflation Data. Based on Table 3, it is known that the p-value is less than α, and the value of $|Z(I)| > Z_{\frac{\alpha}{2}} = 1.96$, which means that the null hypothesis (H_0) is rejected. Therefore, there is a statistically significant spatial autocorrelation in inflation data in five cities in South Sulawesi Province. This shows that inflation in five cities is not only influenced by inflation in the location itself but is also influenced by four other locations.

3.6 Testing the Stationarity of Inflation Data against Variation

Data is considered stationary concerning variance when its variance does not change over time. Data is stationary concerning variance if the λ value from the Box-Cox transformation has a rounded value equal to 1. If the rounded value is not equal to 1, then a transformation is required. The data used to create the Box-Cox plot must have positive values, while some inflation data contain negative values. Therefore, a transformation is needed so that all data becomes positive. The inflation data of five cities in South Sulawesi Province was transformed into:

$$Z_t^* = Z_t + 2 \tag{2}$$

Table 4 displys Box-Cox transformation form of five cities in South Sulawesi Province. Based on the results of the Box-Cox transformation presented in Table 4, Watampone City, Bulukumba City, Makassar City, and Palopo City require one Box-Cox transformation so that the data is stationary concerning the variance, while for Pare-pare City, no transformation was carried out because the data was already stationary concerning the variance before the transformation was carried out.

Table 4. Box-Cox Transformation Form of Five Cities in South Sulawesi Province

Location	λ	Transformation 1 (Z_t^{**}) Transformation Form	λ
Bulukumba	0	lnZ_t^*	1
Watampone	0	lnZ_t^*	1
Makassar	0.5	$\sqrt{Z_t^*}$	1
Pare-pare	1	-	0
Palopo	-0.5	$\frac{1}{\sqrt{Z_t^*}}$	1

3.7 Testing the Stationarity of Inflation Data to the Mean

After the data is stated to be stationary against the variance, the next step is to test the stationarity of the data against the mean. In addition to using the ACF plot, stationarity to the mean can be seen through the Augmented Dickey-Fuller (ADF) test, this test can be done with the following hypothesis.
H_0 : $\phi^* \geq 0$ (data is not stationary to the mean)
H_1 : $\phi^* < 0$ (data is stationary to the mean)

Table 5. Augmented Dickey-Fuller Inflation Data Test Results

Location	$p - value$	$\tau_{statistik}$
Bulukumba	0.01	-4.48
Watampone	0.03	-3.68
Makassar	0.03	-3.74
Pare-pare	0.03	-3.71
Palopo	0.01	-4.24

Table 5 gives details of Augmented Dickey-Fuller inflation data test results. Table 5 shows that the inflation data of five cities in South Sulawesi Province which were previously stationary to the variance, also meet the stationarity to the mean. This can be seen based on the p-value which is smaller than the significance level of 0.05, so it can be concluded that the inflation data of five cities in South Sulawesi Province do not require a differentiation process.

3.8 Identification of GSTARMA Model on Inflation Data

According to Ozcicek & McMilin [20], akaike's Information Criteria Corrected (AICC) can provide a model with a more optimal lag length compared to other indicators. Determining the order in the GSTARMA model can be done by selecting the model that has the smallest AICC value. Table 6 shows this result.

Table 6. Akaike's Information Criteria Values Corrected Inflation Data

Lag	MA (0)	MA (1)	MA (2)	MA (3)	MA (4)
AR (0)	-14.4897	-13.9055	-13.3938	-12.7673	-11.9692
AR (1)	-19.7543	-20.4192	-19.2340	-18.5567	-17.5363
AR (2)	-20.3404	-19.5653	-18.9468	-18.0577	-16.7707
AR (3)	-20.0028	-19.0918	-18.3268	-17.6178	-16.0415
AR (4)	-19.4192	-18.3805	-17.3528	-16.0907	-14.1761
AR (5)	-18.7398	-17.6427	-16.2950	-13.8489	-10.9725
AR (6)	-17.4882	-14.9882	-12.7259	-8.7842	-3.5136

Table 6 shows the AICC values of several autoregressive and moving average orders. Based on the table, the smallest AICC value is obtained at the AR(1) and MA(1) orders, therefore the model that fits the data characteristics is GSTARMA (1,0,1) which has an AR vector of order one and an MA vector of order one and the spatial order for the AR and MA conditions is one. The GSTARMA model formed can be written as follows.

$$Z^{**}_{(t)} = \Phi_{10}W^{(0)}Z^{**}_{(t-1)} + \Phi_{11}W^{(1)}Z^{**}_{(t-1)} + e_t - \Theta_{10}W^{(0)}e_{(t-1)} - \Theta_{11}W^{(1)}e_{(t-1)} \quad (3)$$

with:
$Z^{**}_{(t)}$: the transformed observation data vector at time t
Φ_{10} : diagonal matrix of autoregressive vector parameters at time lag 1 and spatial lag 0
Φ_{11} : diagonal matrix of autoregressive vector parameters at time lag 1 and spatial lag 1
Θ_{10} : diagonal matrix of moving average vector parameters at time lag 1 and spatial lag 0
Θ_{11} : diagonal matrix of moving average vector parameters at time lag 1 and spatial lag 1
$W^{(0)}$: spatial weighting matrix at spatial lag 0
$W^{(1)}$: spatial weighting matrix at spatial lag 1
$e_{(t)}$: vector of error values at time t
$e_{(t-1)}$: vector of error values at time $t-1$

3.9 Calculation of Spatial Location Weights in Inflation Data

Spatial location weights are used to describe the extent to which a city influences other cities in inflation dynamics. In this study, the location weights used are uniform location weights and normalized cross-correlation.

3.9.1 Uniform Location Weighting of Inflation Data

Uniform location weight assumes that the locations analyzed have the same influence by giving the same weight value to all locations. The calculation of uniform location weight can use the following formula.

$$W_{ij} = \frac{1}{n} \quad (4)$$

$$W_{ij} = \begin{cases} \frac{1}{n}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases} \quad (5)$$

with:
n : number of adjacent locations (N-1)
W_ij : uniform location weight from location i to location j

Based on the calculations that have been carried out, the following uniform location weight matrix is obtained:

$$W = \begin{bmatrix} 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 \end{bmatrix}$$

In this matrix, each city gets the same influence, which is 0.25, from the other four cities, because all weights between cities have the same value, this matrix is called a uniform location weight matrix.

3.9.2 Location Weights Normalized Cross Correlation of Inflation Data

These location weights use the results of the cross-correlation between each location with the appropriate lag. This process will produce appropriate location weights that satisfy the equation $\sum_{i\neq j} |W_{ij}| = 1$.

$$W_{ij} = \frac{r_{ij}(k)}{\sum_{k\neq i} |r_{ik}(k)|}, \quad i \neq j \quad (6)$$

Based on the calculations that have been carried out, the location weight matrix is obtained as follows.

$$W = \begin{bmatrix} 0 & 0.27949 & 0.264754 & 0.254867 & -0.20089 \\ 0.122326 & 0 & 0.268712 & 0.384843 & -0.22412 \\ 0.142368 & 0.468307 & 0 & -0.13035 & -0.25898 \\ 0.133388 & 0.198944 & 0.324976 & 0 & -0.34269 \\ -0.14087 & -0.29221 & -0.28109 & -0.28583 & 0 \end{bmatrix}$$

Table 7. OLS Estimation with Uniform Location Weights on Inflation Data

Parameter	Estimated Value	*p − value*	Parameter	Estimated Value	*p − value*
Φ_1^0	-0.57483	0.0331	Θ_1^0	-0.43178	0.0200
Φ_1^1	0.956638	< 0.0001	Θ_1^1	-0.02402	0.9157
Φ_2^0	0.356065	0.0161	Θ_2^0	-0.03413	0.9455
Φ_2^1	-0.63251	0.0003	Φ_1^1	2.483113	0.0423
Φ_3^0	-0.77320	0.0003	Θ_3^0	-0.03891	0.9329
Φ_3^1	1.808780	< 0.0001	Θ_3^1	-0.57836	0.6401
Φ_4^0	0.966328	0.0682	Φ_5^0	0.737177	< 0.0001
Φ_4^1	-1.09234	0.0476	Φ_5^1	0.133177	0.0118
Φ_5^0	1.194967	< 0.0001	Θ_5^0	-0.30042	0.1222
Φ_5^1	-0.26066	0.3303	Θ_5^1	-0.07193	0.1794

The matrix above shows the spatial weights between five cities based on the results of the normalization of the cross-correlation of inflation values. Each element outside the diagonal describes how much influence inflation from one city has on another city. A positive weight value indicates that when inflation in a city increases, inflation in other cities will also increase, while a negative weight value indicates that if inflation in a city increases, inflation in other cities will decrease.

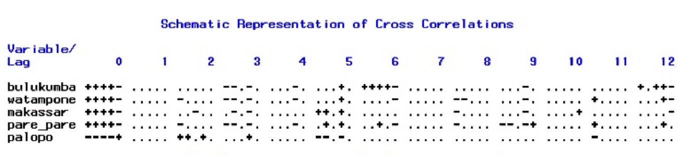

Figure 2. OLS Residual MCCF Plot of Inflation Data

3.10 Parameter Estimation with OLS Method

Parameter estimation using the OLS method is done by minimizing the sum of the squares of the residuals. The GSTARMA model orders used are AR(1) and MA(1) and the spatial order is 1. Table 7 shows the results of parameter estimation using the OLS method with uniform location weights on infation data, while Table 8 displays that estimation with normalized location weights for cross-correlation.

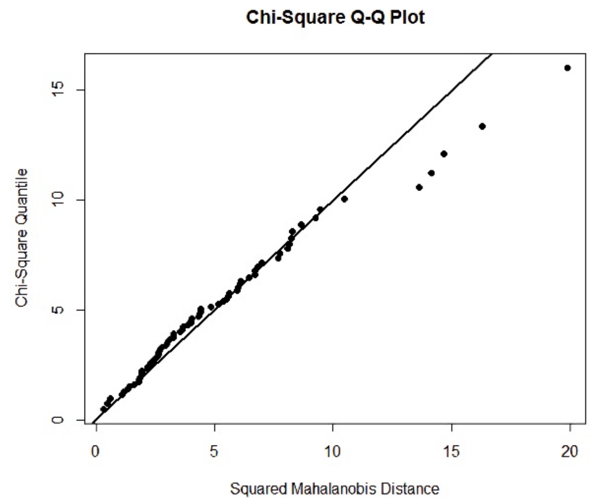

Figure 3. Uniform Location Weighted Normality Plot of Inflation

Table 8. OLS Estimation with Normalized Location Weights for Cross-Correlation on Inflation Data

Parameter	Estimated Value	*p − value*	Parameter	Estimated Value	*p − value*
Φ_1^0	0.145914	0.7124	Θ_1^0	-0.61572	< 0.0001
Φ_1^1	0.632313	0.0361	Θ_1^1	1.268331	0.0061
Φ_2^0	0.002609	0.9934	Θ_2^0	-0.58052	0.1314
Φ_2^1	-0.32070	0.3153	Φ_1^1	6.799812	0.0001
Φ_3^0	-0.26469	0.4282	Θ_3^0	0.578443	0.1227
Φ_3^1	0.818914	0.0020	Θ_3^1	0.813226	< 0.0001
Φ_4^0	0.704610	0.0650	Φ_5^0	-0.09106	0.0127
Φ_4^1	-0.75401	0.0050	Φ_5^1	-0.43436	0.0195
Φ_5^0	1.00650	< 0.0001	Θ_5^0	-1.12199	0.0091
Φ_5^1	-1.18155	0.0087	Θ_5^1	0.05506	0.0812

Based on Table 7 and Table 8, the estimated values of the model parameters obtained using the OLS method can be seen.

In the table, there are several significant and insignificant autoregressive and moving average parameters.

3.11 Parameter Estimation with SUR Method

Estimation of the Seemingly Unrelated Regression (SUR) model can be done using the Generalized Least Square (GLS) method. Figure 2 shows the results obtained from the residual MCCF plot, and it shows that there are several significant lags marked with the symbols (+) and (-).

Based on Figure 2 it can be concluded that the residuals produced by the OLS method have a correlation between equations which causes the OLS method to be less effective in estimating parameters so the SUR method is used. Table 9 and Table 10 display the results of parameter estimation using the SUR method.

Table 9. Estimation of SUR with Uniform Location Weights on Inflation Data

Parameter	Estimated Value	$p-value$	Parameter	Estimated Value	$p-value$
Φ_1^0	-0.09426	0.5736	Θ_1^0	-0.238015	0.2346
Φ_1^1	0.664498	< 0.0001	Θ_1^1	-0.23482	0.0783
Φ_2^0	0.089962	0.5445	Θ_2^0	-0.37320	0.0371
Φ_2^1	-0.46553	0.0001	Φ_1^1	1.682428	0.0490
Φ_3^0	-0.09909	0.6659	Θ_3^0	-0.23883	0.3877
Φ_3^1	0.669632	< 0.0001	Θ_3^1	-0.17296	0.8468
Φ_4^0	0.273184	0.2206	Φ_5^0	0.918074	< 0.0001
Φ_4^1	-0.60236	0.0170	Φ_5^1	0.042499	0.2619
Φ_5^0	0.817601	< 0.0001	Θ_5^0	-0.48031	0.0008
Φ_5^1	-0.23801	0.2346	Θ_5^1	0.002286	0.9585

Based on Table 9 and Table 10, the estimated values of the model parameters obtained using the SUR method can be seen. In the table, there are several significant and insignificant autoregressive and moving average parameters. Although there are several insignificant parameters, these parameters are still included in the model because they consider the influence of the weight of each region. Therefore, in this study, insignificant parameters are still used in the forecast.

3.12 GSTARMA Model Diagnostic Testing

Model diagnostic tests are conducted to assess the feasibility of a model in forecasting. Two assumptions required in model

Table 10. Estimation of SUR with Normalized Location Weights of Cross-Correlation on Inflation Data

Parameter	Estimated Value	$p-value$	Parameter	Estimated Value	$p-value$
Φ_1^0	0.627008	0.0116	Θ_1^0	-0.104348	< 0.0001
Φ_1^1	0.273347	0.1372	Θ_1^1	0.055228	0.8766
Φ_2^0	-0.45893	0.6644	Θ_2^0	0.070420	0.7799
Φ_2^1	0.041115	0.8384	Φ_1^1	3.989007	0.0005
Φ_3^0	0.395338	0.0465	Θ_3^0	-0.04434	0.8598
Φ_3^1	0.390306	0.0035	Θ_3^1	-1.13174	0.3390
Φ_4^0	-0.06439	0.7590	Φ_5^0	1.008778	< 0.0001
Φ_4^1	-0.22888	0.1442	Φ_5^1	0.001229	0.9653
Φ_5^0	0.990792	< 0.0001	Θ_5^0	-0.53324	0.0004
Φ_5^1	0.114199	0.2663	Θ_5^1	-0.00377	0.8929

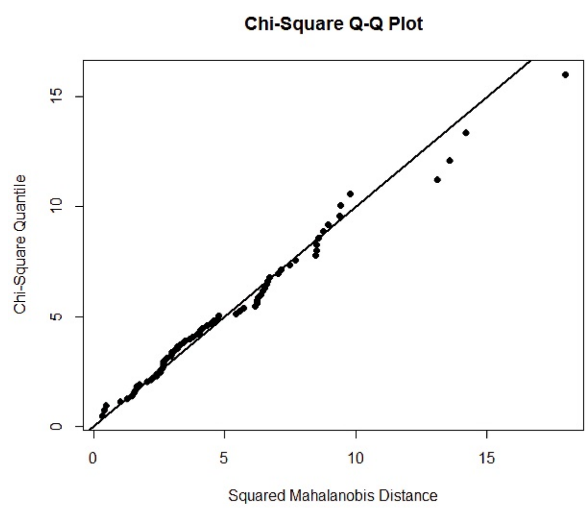

Figure 4. The plot of Normality of Location Weights Normalized Cross-Correlation of Inflation

diagnostic tests are that the residuals are white noise and follow a multivariate normal distribution.

3.12.1 Multivariate Normality Test for Inflation Data

The normality test is carried out to assess whether the residuals of the model are normally distributed or not. The multivariate normality test can be done by looking at the $q-q$ plot. Data is said to be normally distributed if the residual plot spreads around the diagonal line. Figure 3 presents the residual plot of the GSTARMA model with uniform location weights, while Figure 4 presents the cross-correlation normalization.

Based on Figure 3 and Figure 4, the residual plot of the GSTARMA model with uniform location weights and cross-correlation normalization shows a pattern that tends to follow and is around the diagonal line. Therefore, it can be concluded that the residual plot of the GSTARMA model is distributed normally multivariately.

Table 11. Results of the Portmanteau Test with Uniform Location Weights for Inflation Data

Lag	$\chi^2_{N^2(k)}$	Q	$p - value$
2	37.65	25.03	0.4605
3	67.50	48.61	0.5292
4	96.22	81.64	0.2806
5	124.34	123.54	0.0557
6	152.09	147.63	0.0815

3.12.2 White Noise Test of Inflation Data

The assumption that the residuals are white noise can be shown by looking at the $p - value$ of the portmanteau test as follows. Table 11 and Table 12 show the results.

Table 12. Portmanteau Test Results with Normalized Location Weights for Cross-Correlation of Inflation Data

Lag	$\chi^2_{N^2(k)}$	Q	$p - value$
2	37.65	25.25	0.4486
3	67.50	46.47	0.6158
4	96.22	77.77	0.3907
5	124.34	123.10	0.0584
6	152.09	146.28	0.0938

Based on Table 11 and Table 12, it can be seen that the p-value is greater than the value of $\alpha = 0.05$ and the value of $Q < \chi^2_{N^2(k)}$, which means that H_0 is not rejected. Therefore, it can be concluded that the residuals of the GSTARMA model have met the white noise assumption.

3.13 Selecting the Best Model for Inflation Data

The selection of the best GSTARMA model is done by looking at the smallest RMSE value of the model formed. Table 13 shows the results.

Based on Table 13, it is known that the smallest RMSE value is obtained from the model with cross-correlation normalization location weights, therefore the best GSTARMA model is the one that uses cross-correlation normalization location weights.

3.14 Inflation Rate Forecasting

To obtain forecasting results in the GSTARMA model, a back transformation is required because the data used for modeling and forecasting are Box-Cox transformed data. Table 14 displays the form of the back transformation of each data at each location.

Table 13. Results of Calculation of RMSE Value of Inflation Data

Location	Uniform Location Weights	Normalized Location Weights Cross-Correlation
Bulukumba	0.1557	0.1288
Watampone	0.2408	0.2540
Makassar	0.2656	0.2259
Pare-pare	0.2578	0.2555
Palopo	0.2655	0.2836

Table 14. Back Transformation of Inflation Data at Each Location

Location	Transformation Form	Reverse Transformation
Bulukumba	$\ln Z_t^*$	$e^{Z_t^{**}} - 2$
Watampone	$\ln Z_t^*$	$e^{Z_t^{**}} - 2$
Makassar	$\sqrt{Z_t^*}$	$(Z_t^{**})^2 - 2$
Pare-pare	-	$Z_t^* - 2$
Palopo	$\frac{1}{\sqrt{Z_t^*}}$	$\frac{1}{Z_t^{**2}} - 2$

3.15 Forecasting Using the Best Model

After getting the best model based on the smallest RMSE value, the next step is to forecast the inflation value for the next few periods. Figure 5 shows the results of the inflation value forecast for each location, and Table 13 gives the detail.

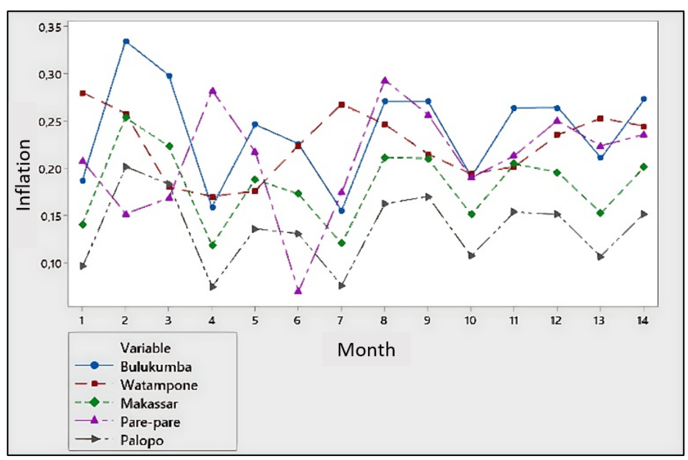

Figure 5. Inflation Forecast Plot of Five Cities in South Sulawesi Province

Figure 5 shows the results of inflation forecasting in five cities in South Sulawesi Province with a relatively similar pattern, although there are still fluctuations between months. In general, data movements in each city follow a similar pattern without any high spikes in one particular city. This shows that the forecasting model captures a stable pattern in the five cities analyzed.

Table 15. Inflation Value Forecasting

Month	Bulukumba	Watampone	Makassar	Parepare	Palopo
Nov 2024	0.1869	0.2803	0.1413	0.2080	0.0965
Dec 2024	0.3343	0.2583	0.2536	0.1522	0.2018
Jan 2025	0.2985	0.1807	0.2237	0.1689	0.1834
Feb 2025	0.1594	0.1705	0.1190	0.2827	0.0750
Mar 2025	0.2463	0.1763	0.1886	0.2176	0.1364
Apr 2025	0.2260	0.2242	0.1732	0.0700	0.1309
May 2025	0.1557	0.2680	0.1208	0.1743	0.0764
Jun 2025	0.2707	0.2465	0.2118	0.2935	0.1627
Jul 2025	0.2707	0.2153	0.2105	0.2571	0.1704
Aug 2025	0.1912	0.1942	0.1514	0.1902	0.1074
Sep 2025	0.2637	0.2020	0.2050	0.2136	0.1538
Oct 2025	0.2644	0.2355	0.1959	0.2507	0.1517
Nov 2025	0.2112	0.2530	0.1531	0.2240	0.1072
Dec 2025	0.2730	0.2448	0.2017	0.2359	0.1521

Table 15 shows the inflation forecast values for the next 14 periods. The highest inflation prediction value in Bulukumba City occurred in December 2024, while Watampone City had the highest inflation prediction in November 2024. Makassar City had the highest inflation prediction in December 2024. Pare-pare City had the highest inflation prediction in June 2025 and Palopo City had the highest inflation prediction in December 2024.

4. CONCLUSIONS

Based on the findings of the analysis, we conclude that the appropriate GSTARMA model for predicting inflation in five cities is GSTARMA (1,0,1) with the order of the AR vector, MA vector, and spatial order equal to one. The appropriate location weights for the GSTARMA model in this analysis are the normalized location weights of cross-correlation. Moreover, the inflation in five cities is influenced by inflation in the previous period and inflation in one city is influenced by inflation in four other cities in the previous period.

5. ACKNOWLEDGEMENT

We would like to express our deepest gratitude to all lecturers, staff, and friends who have provided support, direction, and knowledge that has been very helpful in this research.

REFERENCES

[1] G. E. P. Box, G. M. Jenkins, G. C. Reinsel, and G. M. Ljung. *Time Series Analysis: Forecasting and Control.* John Wiley & Sons, Hoboken, 5th edition, 2016.

[2] W. W. S. Wei. *Time Series Analysis: Univariate and Multivariate Methods.* Pearson Education, USA, 2nd edition, 2006.

[3] D. Munandar, B. N. Ruchjana, A. S. Abdullah, and H. F. Pardede. Integration gstarima with deep neural network to enhance prediction accuracy on rainfall data. *Systems Science & Control Engineering*, 12(1), 2024.

[4] A. B. Salsabila, B. N. Ruchjana, and A. S. Abdullah. Development of the gstarima (1,1,1) model order for climate data forecasting. *International Journal of Data and Network Science*, 8:773–788, 2024.

[5] A. Safira, R. A. Dhiya'ulhaq, I. Fahmiyah, and M. Ghani. Spatial impact on inflation of java island prediction using autoregressive integrated moving average (arima) and generalized space-time arima (gstarima). *MethodsX*, 13:102867, 2024.

[6] P. Monika, B. N. Ruchjana, A. S. Abdullah, and R. Budiarto. Integration of gstarima model with heteroskedastic error and kriging for climate forecasting: A systematic review. *Applied Mathematics & Information Sciences: An International Journal*, 18(3):551–567, 2024.

[7] N. Aulia and D. R. S. Saputro. Generalized space time autoregressive integrated moving average with exogenous (gstarima-x) models. In *Journal of Physics: Conference Series*, volume 1808, page 012052, 2021.

[8] M. S. Akbar, Setiawan, Suhartono, B. N. Ruchjana, D. D. Prastyo, A. Muhaimin, and E. Setyowati. A generalized space-time autoregressive moving average (gstarma) model for forecasting air pollutant in surabaya. In *Journal of Physics: Conference Series*, volume 1490, page 012022, 2020.

[9] S. Ajobo, O. Alaba, and A. Zaenal. Generalised space-time seasonal autoregressive integrated moving average seemingly unrelated regression modelling of seasonal and nonstationary data. *Scientific African*, 24:e02189, 2024.

[10] P. Monika, B. N. Ruchjana, and A. S. Abdullah. Gstari-x-arch model with data mining approach for forecasting climate in west java. *Computation*, 10(12):204, 2022.

[11] M. R. Kurniawan, U. Mukhaiyar, and S. W. Indratno. Modelling of data following a gstarma-x model using kalman filter. In *AIP Conference Proceedings*, volume 2774, 2024.

[12] U. Mukhaiyar, A. W. Mahdiyasah, and T. Prastoro. The generalized space-time autoregressive modelling with three dimensional of spatial weight matrix in predicting the indonesia peatland's water level. *Environmental Science Europe*, 36:180, 2024.

[13] U. S. Pasaribu, U. Mukhaiyar, M. N. Heriawan, and Y. Yundari. Generalized space-time autoregressive modeling of the vertical distribution of copper and gold grades with a porphyry-deposit case study. *International Journal on Advanced Science Engineering Information Technology*, 12(5):2030–2038, 2022.

[14] U. Mukhaiyar, B. I. Bilad, and S. Pasaribu. The generalized space-time autoregressive modeling with minimum spanning tree approach of weight matrix for covid-19 case in java island. In *Journal of Physics: Conference Series*, volume 2084, page 012003, 2021.

[15] U. Mukhaiyar, F. T. Nabilah, U. S. Pasaribu, and N. M. Huda. The space-time autoregressive modelling with time correlated errors for the number of vehicles in purbaleunyi toll gates. In *Journal of Physics: Conference Series*, volume 2243, page 012068, 2022.

[16] N. M. Huda and N. Imro'ah. Determination of the best weight matrix for the generalized space-time model in the covid-19 case on java island. *Spatial Statistics*, 54:100734, 2023.

[17] F. Hestuningtias and M. H. S. Kurniawan. The implementation of the generalized space-time autoregressive model for inflation prediction. *International Journal of Applied Statistics and Data Science*, 3(2):176–188, 2023.

[18] N. M. Huda, U. Mukhaiyar, and U. S. Pasaribu. The approximation of generalized space-time autoregressive model for discreate cases through inar model. In *Journal of Physics: Conference Series*, volume 1722, page 012100, 2021.

[19] J. Lee and D. W. S. Wong. *Statistical Analysis Arcview GIS*. John Wiley and Sons, New York, 2001.

[20] O. Ozcicek and W. McMilin. Lag length selection in vector autoregressive models symmetric and asymmetric lags. *Applied Economics*, 31:517–524, 1999.