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Research Paper

Algebraic Construction of Rough Semimodules Over Rough Rings

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Rough Set, Rough Semimodule, Rough Semiring

Abstract

Let $(\mho, \mu)$ be an approximation space, where $\mho$ is a non-empty set and μ is an equivalence relation on $\mho$. For any subset $H \subseteq \mho$, we can define the lower approximation and the upper approximation of H. A set H is called a rough set if its lower and upper approximations are not equal. In this study, we explore the algebraic structure that emerges when certain binary operations are defined on rough sets. Specifically, we investigate the conditions under which a subset H forms a rough semimodule over a rough semiring. We present several key properties of this structure and construct illustrative examples to support our theoretical results.

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1. INTRODUCTION

Rough set theory is a mathematical technique that was first introduced by Pawlak [1]. This theory is related to the approximation space concept [2]. Given an approximation space $(\mho, R)$ with a non-empty set $\mho$ and R the equivalence relation of U. If X is a subset of U, then the combination of equivalence classes contained in X is called lower approximation and is denoted by $\underline{\text{Apr}}(X)$. In addition, the combination of equivalence classes that intersect the set X which is a non-empty set is called an upper approximation, denoted by $\overline{\text{Apr}}(X)$. A subset X is a rough set if its lower and upper approximations are not equal.

Various studies have been carried out regarding the application of rough sets to algebraic structures. Biswas and Nanda [3] conducted research on rough groups and rough subgroups with both tied to the upper approximation but not to the lower approximation. On the other hand, Wang and Chen [4] also studied about some properties of rough groups. Furthermore, Davvas [5] conducted a study regarding rough sets in rings. Research was also carried out by Davvaz and Mahdavipour [6] regarding rough modules. In 2001, Han [7] studied homomorphism and isomorphism in rough sets. Then, Miao et al. [8] studied rough groups, rough subgroups, and their properties. Previous workers, Qun-Feng et al. [9] studied rough modules and some of their properties. In 2014, it was strengthened by Sinha and Prakash in their research on projective modules in rough sets [10], injective modules based on rough set theory [11], and rough exact sequences of modules [12]. In 2015, Bagirmaz and Ozcan [13] studied rough semigroups on approximation spaces. In 2021, Davvas et al. [14] studied fuzzy sets and rough sets. In 2022, Hafifulloh et al. [15], studied the properties of rough V-coexact sequences in a rough group, and in the same year, Nugraha et al. [16] studied the implementation of a rough set on a group structure. Furthermore, Ayuni et al. [17] studied the rough U-exact sequence of rough groups. Then in 2023, Yanti et al. [18] studied the implementation of a rough set of projective modules. Some researchers also study the rough sets on the structure of semirings, i.e., Praba et al. [19], Manimaran et al. [20], and Selvan and Kumar [21]. Recently, no researcher has constructed a rough semimodule over a rough semiring, so in this research, we try to construct it and provide some properties related to the rough semimodule over a rough semiring.

2. METHODS

This research was conducted through a literature study, involving the collection and analysis of relevant materials from scholarly references such as journals and academic books. The methodological foundation is based on the concepts of upper and lower approximation spaces, rough semimodules, rough semirings, and rough subsemimodules.

The primary objectives of this study are to define and explore the algebraic structures of rough semimodules over rough semirings, to investigate the properties of their substructures, and to construct illustrative examples. The research is carried out through the following stages:

1. We formulate the definition of a semimodule over a rough semiring within the framework of an approximation space.
2. We examine the structural properties of rough subsemimodules over rough semirings in the same context.
3. We analyze the conditions under which the intersection of rough subsemimodules forms a subsemimodule of a rough semimodule.
4. We construct specific examples to illustrate and validate the theoretical properties established in the previous stages.

3. RESULTS AND DISCUSSION

Given an approximation space $(\mho, \mu)$, where $\mho$ is a non-empty set and μ is an equivalence relation on $\mho$. We give the definition of rough semimodule over rough semiring as follows.

Definition 1. Given an approximation space $(\mho, \mu)$, a rough semiring S, and a monoid commutative group M. M is called a rough left semimodule over rough semiring S if there is a mapping

$$\cdot\, : \operatorname{Apr}(S) \times \operatorname{Apr}(M) \to \operatorname{Apr}(M), \quad (r, m) \mapsto rm$$

such that:

1. $r(m_1+m_2) = rm_1+rm_2$, for every $r \in \operatorname{Apr}(S)$, and $m_1, m_2 \in \operatorname{Apr}(M)$;
2. $(r_1 + r_2)m = r_1m + r_2m$, for every $r_1, r_2 \in \operatorname{Apr}(S)$, and $m \in \operatorname{Apr}(M)$;
3. $(r_1r_2)m = r_1(r_2m)$, for every $r_1, r_2 \in \operatorname{Apr}(S)$, and $m \in \operatorname{Apr}(M)$;
4. $1m = m$, where 1 is a unit element of S, and $m \in \operatorname{Apr}(M)$.

A rough right *semimodule* over rough semiring S can be defined similarly. In the following proposition, we give sufficient conditions for a set to be a rough subsemimodule over a rough semiring.

Proposition 1. Given an approximation space $(\mho, \mu)$ and Y is a rough semimodule of rough semiring X in U. If $S \subseteq Y$ with $\operatorname{Apr}(S) = T$, where T is a submodule of Y, then S is a rough subsemimodule of Y.

Proof.

1. Given arbitrary $a, b \in S \subseteq T$. Since T is a subsemimodule of Y, we have $a + b \in T$. Then $a + b \in \operatorname{Apr}(S)$. Hence, we obtain that $a + b \in \operatorname{Apr}(S)$, for every $a, b \in S$.
2. Given arbitrary $r \in X$, $a \in S \subseteq T$. Since T is a subsemimodule of Y, we obtain $ra \in T$ and hence $r \cdot a \in \operatorname{Apr}(S)$. From this result, we have S is a rough subsemimodule of rough semiring X in approximation space $(\mho, \mu)$. ■

The following example is given as an illustration of Proposition 1.

Example 1. Given a universal set $(\mho, \mu)$ with $\mho = \mathbb{Z}_{20}$. The relation μ on set $\mho$ was defined for every $a\mu b$ if $a - b = 6k$ for $k \in \mathbb{Z}$. For equivalence classes in the set $\mho$, the equivalence relation μ is as follows:

$$V_1 = [\bar{0}] = \{\bar{0}, \bar{6}, \overline{12}, \overline{18}\}$$
$$V_2 = [\bar{1}] = \{\bar{1}, \bar{7}, \overline{13}, \overline{19}\}$$
$$V_3 = [\bar{2}] = \{\bar{2}, \bar{8}, \overline{14}\}$$
$$V_4 = [\bar{3}] = \{\bar{3}, \bar{9}, \overline{15}\}$$
$$V_5 = [\bar{4}] = \{\bar{4}, \overline{10}, \overline{16}\}$$
$$V_6 = [\bar{5}] = \{\bar{5}, \overline{11}, \overline{17}\}$$

Now, we will give an example of rough semiring. Given a non-empty subset of set $\mho$, for example, $X \subseteq \mho$, where $X = \{\bar{0}, \bar{1}, \bar{6}, \bar{9}, \overline{10}, \overline{11}, \overline{14}, \overline{19}\}$. Based on equivalence classes in approximation space of $(\mho, \mu)$ that has been known previously, lower and upper approximations of X are obtained as follows:

$$\underline{\operatorname{Apr}}(X) = \{x \mid [x]_R \subseteq X\} = \varnothing;$$

$$\overline{\operatorname{Apr}}(X) = \{x \mid [x]_R \cap X \neq \varnothing\} = \mho$$

To prove that X is a rough semiring, we will use the following Cayley table in Table 1.

Table 1. Cayley Table of X

$+_{20}$	$\bar{0}$	$\bar{1}$	$\bar{6}$	$\bar{9}$	$\overline{10}$	$\overline{11}$	$\overline{14}$	$\overline{19}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{6}$	$\bar{9}$	$\overline{10}$	$\overline{11}$	$\overline{14}$	$\overline{19}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{7}$	$\overline{10}$	$\overline{11}$	$\overline{12}$	$\overline{15}$	$\bar{0}$
$\bar{6}$	$\bar{6}$	$\bar{7}$	$\overline{12}$	$\overline{15}$	$\overline{16}$	$\overline{17}$	$\bar{0}$	$\bar{5}$
$\bar{9}$	$\bar{9}$	$\overline{10}$	$\overline{15}$	$\overline{18}$	$\overline{19}$	$\bar{0}$	$\bar{3}$	$\bar{8}$
$\overline{10}$	$\overline{10}$	$\overline{11}$	$\overline{16}$	$\overline{19}$	$\bar{0}$	$\bar{1}$	$\bar{4}$	$\bar{9}$
$\overline{11}$	$\overline{11}$	$\overline{12}$	$\overline{17}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{5}$	$\overline{10}$
$\overline{14}$	$\overline{14}$	$\overline{15}$	$\bar{0}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{8}$	$\overline{13}$
$\overline{19}$	$\overline{19}$	$\bar{0}$	$\bar{5}$	$\bar{8}$	$\bar{9}$	$\overline{10}$	$\overline{13}$	$\overline{18}$

Based on Table 1, it is proven that:

1. For every $x, y \in X$, $x +_{20} y \in \overline{\operatorname{Apr}}(X)$;
2. For every $x, y, z \in X$, $(x +_{20} y) +_{20} z = x +_{20} (y +_{20} z)$ applies in $\overline{\operatorname{Apr}}(X)$;
3. There is $\bar{0} \in \overline{\operatorname{Apr}}(X)$, therefore applies $x +_{20} \bar{0} = \bar{0} +_{20} x = x$;
4. For every $x, y \in X$ applies $x +_{20} y = y +_{20} x$, therefore, X with operation $+_{20}$ is commutative.

Now, we will check for the operation of multiplication modulo 20 in X. The following Cayley table for multiplication modulo 20 in X is presented in Table 2.

5. $x \cdot_{20} y \in \overline{\operatorname{Apr}}(X)$, for every $x, y \in X$;
6. $(x \cdot_{20} y) \cdot_{20} z = x \cdot_{20} (y \cdot_{20} z)$ satisfied in $\overline{\operatorname{Apr}}(X)$, for every $x, y, z \in X$;
7. The left distributive and right distributive laws hold in $\overline{\operatorname{Apr}}(X)$.

Table 2. Cayley Table Operation $\cdot_{20}$

$\cdot_{20}$	$\overline{0}$	$\overline{1}$	$\overline{6}$	$\overline{9}$	$\overline{10}$	$\overline{11}$	$\overline{14}$	$\overline{19}$
$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$
$\overline{1}$	$\overline{0}$	$\overline{1}$	$\overline{6}$	$\overline{9}$	$\overline{10}$	$\overline{11}$	$\overline{14}$	$\overline{19}$
$\overline{6}$	$\overline{0}$	$\overline{6}$	$\overline{16}$	$\overline{14}$	$\overline{0}$	$\overline{6}$	$\overline{4}$	$\overline{14}$
$\overline{9}$	$\overline{0}$	$\overline{9}$	$\overline{14}$	$\overline{1}$	$\overline{10}$	$\overline{19}$	$\overline{6}$	$\overline{11}$
$\overline{10}$	$\overline{0}$	$\overline{10}$	$\overline{0}$	$\overline{10}$	$\overline{0}$	$\overline{10}$	$\overline{0}$	$\overline{10}$
$\overline{11}$	$\overline{0}$	$\overline{11}$	$\overline{6}$	$\overline{19}$	$\overline{10}$	$\overline{1}$	$\overline{14}$	$\overline{9}$
$\overline{14}$	$\overline{0}$	$\overline{14}$	$\overline{4}$	$\overline{6}$	$\overline{0}$	$\overline{14}$	$\overline{16}$	$\overline{6}$
$\overline{19}$	$\overline{0}$	$\overline{19}$	$\overline{14}$	$\overline{11}$	$\overline{10}$	$\overline{9}$	$\overline{6}$	$\overline{1}$

From (1–7), it is proven that X is a rough semiring of approximation space $(\mho, \mu)$. After rough semiring in approximation space is obtained, in the following, an example of rough semimodule will be constructed using the approximation space in Example 1.

Example 2. Given $Y \subseteq \mho$, where $Y = \{\overline{0}, \overline{1}, \overline{2}, \overline{4}, \overline{5}, \overline{8}, \overline{10}, \overline{12}, \overline{15}, \overline{16}, \overline{18}, \overline{19}\}$. In a known approximation space $(\mho, \mu)$, lower and upper approximation of subset Y was determined. The lower and upper approximations of Y are:

$$\underline{\text{Apr}}(Y) = \{y \mid [y]_R \subseteq Y\} = V_5 = \{\overline{4}, \overline{10}, \overline{16}\}$$

$$\overline{\text{Apr}}(Y) = \{y \mid [y]_R \cap Y \neq \varnothing\} = \mho$$

The following Cayley table of Y was constructed to show that the set is contained in an upper approximation of Y.

Table 3. Cayley Table Operation $+_{20}$ in Y

$+_{20}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{4}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{12}$	$\overline{15}$	$\overline{16}$	$\overline{18}$	$\overline{19}$
$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{4}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{12}$	$\overline{15}$	$\overline{16}$	$\overline{18}$	$\overline{19}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{5}$	$\overline{6}$	$\overline{9}$	$\overline{11}$	$\overline{13}$	$\overline{16}$	$\overline{17}$	$\overline{19}$	$\overline{0}$
$\overline{2}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{6}$	$\overline{7}$	$\overline{10}$	$\overline{12}$	$\overline{14}$	$\overline{17}$	$\overline{18}$	$\overline{0}$	$\overline{1}$
$\overline{4}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	$\overline{8}$	$\overline{9}$	$\overline{12}$	$\overline{14}$	$\overline{16}$	$\overline{19}$	$\overline{0}$	$\overline{2}$	$\overline{3}$
$\overline{5}$	$\overline{5}$	$\overline{6}$	$\overline{7}$	$\overline{9}$	$\overline{10}$	$\overline{13}$	$\overline{15}$	$\overline{17}$	$\overline{0}$	$\overline{1}$	$\overline{3}$	$\overline{4}$
$\overline{8}$	$\overline{8}$	$\overline{9}$	$\overline{10}$	$\overline{12}$	$\overline{13}$	$\overline{16}$	$\overline{18}$	$\overline{0}$	$\overline{3}$	$\overline{4}$	$\overline{6}$	$\overline{7}$
$\overline{10}$	$\overline{10}$	$\overline{11}$	$\overline{12}$	$\overline{14}$	$\overline{15}$	$\overline{18}$	$\overline{0}$	$\overline{2}$	$\overline{5}$	$\overline{6}$	$\overline{8}$	$\overline{9}$
$\overline{12}$	$\overline{12}$	$\overline{13}$	$\overline{14}$	$\overline{16}$	$\overline{17}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{7}$	$\overline{8}$	$\overline{10}$	$\overline{11}$
$\overline{15}$	$\overline{15}$	$\overline{16}$	$\overline{17}$	$\overline{19}$	$\overline{0}$	$\overline{3}$	$\overline{5}$	$\overline{7}$	$\overline{10}$	$\overline{11}$	$\overline{13}$	$\overline{14}$
$\overline{16}$	$\overline{16}$	$\overline{17}$	$\overline{18}$	$\overline{0}$	$\overline{1}$	$\overline{4}$	$\overline{6}$	$\overline{8}$	$\overline{11}$	$\overline{12}$	$\overline{14}$	$\overline{15}$
$\overline{18}$	$\overline{18}$	$\overline{19}$	$\overline{0}$	$\overline{2}$	$\overline{3}$	$\overline{6}$	$\overline{8}$	$\overline{10}$	$\overline{13}$	$\overline{14}$	$\overline{16}$	$\overline{17}$
$\overline{19}$	$\overline{19}$	$\overline{0}$	$\overline{1}$	$\overline{3}$	$\overline{4}$	$\overline{7}$	$\overline{9}$	$\overline{11}$	$\overline{14}$	$\overline{15}$	$\overline{17}$	$\overline{18}$

Based on Table 3, it is proven that:

1. For every $x, y \in Y$, implies $x +_{20} y \in \overline{\text{Apr}}(Y)$;
2. For every $x, y, z \in Y$, $(x +_{20} y) +_{20} z = x +_{20} (y +_{20} z)$ satisfied in $\overline{\text{Apr}}(Y)$;
3. There is $\overline{0} \in \overline{\text{Apr}}(Y)$, therefore for every $x \in Y$ implies $x +_{20} \overline{0} = \overline{0} +_{20} x = x$;
4. For every $x, y \in Y$ implies $x +_{20} y = y +_{20} x$, therefore Y with operation $+_{20}$ is commutative.

Therefore, it is proven that Y is a rough monoid.

Next is to determine that Y is the upper rough semimodule of rough semiring X. The following is Cayley's table of scalar multiplication of upper rough semimodule of rough semiring.

Based on Table 3 and Table 4, it was obtained:

Table 4. Cayley Table for Scalar Multiplication in Y

	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{4}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{12}$	$\overline{15}$	$\overline{16}$	$\overline{18}$	$\overline{19}$
$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$
$\overline{1}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{4}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{12}$	$\overline{15}$	$\overline{16}$	$\overline{18}$	$\overline{19}$
$\overline{6}$	$\overline{0}$	$\overline{6}$	$\overline{12}$	$\overline{4}$	$\overline{10}$	$\overline{8}$	$\overline{0}$	$\overline{12}$	$\overline{10}$	$\overline{16}$	$\overline{8}$	$\overline{14}$
$\overline{9}$	$\overline{0}$	$\overline{9}$	$\overline{18}$	$\overline{16}$	$\overline{15}$	$\overline{12}$	$\overline{10}$	$\overline{8}$	$\overline{15}$	$\overline{2}$	$\overline{4}$	$\overline{11}$
$\overline{10}$	$\overline{0}$	$\overline{10}$	$\overline{0}$	$\overline{10}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{10}$
$\overline{11}$	$\overline{0}$	$\overline{11}$	$\overline{2}$	$\overline{4}$	$\overline{15}$	$\overline{10}$	$\overline{12}$	$\overline{5}$	$\overline{16}$	$\overline{18}$	$\overline{9}$	$\overline{3}$
$\overline{14}$	$\overline{0}$	$\overline{14}$	$\overline{8}$	$\overline{16}$	$\overline{10}$	$\overline{12}$	$\overline{0}$	$\overline{14}$	$\overline{2}$	$\overline{4}$	$\overline{12}$	$\overline{6}$
$\overline{19}$	$\overline{0}$	$\overline{19}$	$\overline{18}$	$\overline{16}$	$\overline{15}$	$\overline{12}$	$\overline{10}$	$\overline{8}$	$\overline{5}$	$\overline{4}$	$\overline{2}$	$\overline{1}$

1. $a \cdot_{20} (x +_{20} y) = (a \cdot_{20} x) +_{20} (a \cdot_{20} y)$, for every $a \in X$ and $x, y \in Y$.
2. $(a +_{20} b) \cdot_{20} x = (a \cdot_{20} x) +_{20} (b \cdot_{20} x)$, for every $a, b \in X$ and $x \in Y$.
3. $(a \cdot_{20} b) \cdot_{20} x = a \cdot_{20} (b \cdot_{20} x)$, for every $a, b \in X$ and $x \in Y$.
4. There is $\overline{0} \in \overline{\text{Apr}}(X)$, therefore for every $x \in Y$ implies $\overline{1} \cdot_{20} x = x$, with $\overline{1}$ as a unit element of X.

It is proven that Y is the upper rough semimodule of rough semiring X.

Example 3. Given a non-empty set $Y \subseteq \mho$ with $Y = \{\overline{0}, \overline{1}, \overline{2}, \overline{4}, \overline{5}, \overline{8}, \overline{10}, \overline{12}, \overline{15}, \overline{16}, \overline{18}, \overline{19}\}$, if we choose $S' = \{\overline{0}, \overline{1}, \overline{5}, \overline{8}, \overline{10}, \overline{15}\}$, then we obtain: $\underline{\text{Apr}}(S') = \{x \mid [x]_R \subseteq S'\} = \varnothing$; $\overline{\text{Apr}}(S') = \{x \mid [x]_R \cap S' \neq \varnothing\} = \overline{\mho}$.

Now, we will check that S' is a rough subsemimodule of Y:

1. Given any $a, b \in S'$, Table 5 shows that $a +_{20} b \in \overline{\text{Apr}}(S')$.

Table 5. Cayley Table Operation $+_{20}$ in S'

$+_{20}$	$\overline{0}$	$\overline{1}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{15}$
$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{15}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{6}$	$\overline{9}$	$\overline{11}$	$\overline{16}$
$\overline{5}$	$\overline{5}$	$\overline{6}$	$\overline{10}$	$\overline{13}$	$\overline{15}$	$\overline{0}$
$\overline{8}$	$\overline{8}$	$\overline{9}$	$\overline{13}$	$\overline{16}$	$\overline{18}$	$\overline{3}$
$\overline{10}$	$\overline{10}$	$\overline{11}$	$\overline{15}$	$\overline{18}$	$\overline{0}$	$\overline{5}$
$\overline{15}$	$\overline{15}$	$\overline{16}$	$\overline{0}$	$\overline{3}$	$\overline{5}$	$\overline{10}$

2. Given any $r \in X$, $a \in S'$, Table 6 shows that $r \cdot_{20} a \in \overline{\text{Apr}}(S')$.

Table 6. Cayley Table for Scalar Multiplication

$\cdot_{20}$	$\overline{0}$	$\overline{1}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{15}$
$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$
$\overline{1}$	$\overline{0}$	$\overline{1}$	$\overline{5}$	$\overline{8}$	$\overline{10}$	$\overline{15}$
$\overline{6}$	$\overline{0}$	$\overline{6}$	$\overline{10}$	$\overline{8}$	$\overline{12}$	$\overline{10}$
$\overline{9}$	$\overline{0}$	$\overline{9}$	$\overline{5}$	$\overline{12}$	$\overline{10}$	$\overline{15}$
$\overline{10}$	$\overline{0}$	$\overline{10}$	$\overline{10}$	$\overline{0}$	$\overline{0}$	$\overline{10}$
$\overline{11}$	$\overline{0}$	$\overline{11}$	$\overline{15}$	$\overline{8}$	$\overline{10}$	$\overline{5}$
$\overline{14}$	$\overline{0}$	$\overline{14}$	$\overline{0}$	$\overline{12}$	$\overline{0}$	$\overline{10}$
$\overline{19}$	$\overline{0}$	$\overline{19}$	$\overline{15}$	$\overline{12}$	$\overline{10}$	$\overline{5}$

Therefore, it can be concluded that S' is a rough subsemimodule of rough semimodule Y over rough semiring X.

In the following proposition, we will show that the properties of the intersection of rough subsemimodules.

Proposition 2. Given an approximation space $(\mho, \mu)$ and Y is a rough semimodule of rough semiring X in $\mho$. If $S_1, S_2, \ldots, S_n$ are rough subsemimodules of Y over rough semiring X, then $\bigcap_{i=1}^{n} S_i$ is also a rough subsemimodule of rough semimodule Y if $\bigcap_{i=1}^{n} \overline{\text{Apr}}\ S_i = \overline{\text{Apr}}\ \bigcap_{i=1}^{n} S_i$.

Proof.

1. Given arbitrary $a, b \in \bigcap_{i=1}^{n} S_i$. By assumption, $S_1, S_2, \ldots, S_n$ are rough subsemimodules of Y over rough semiring X. Therefore, we obtain $a, b \in \bigcap_{i=1}^{n} S_i \subseteq S_i$, for every $i = 1, 2, \ldots, n$. Since S_i is a rough subsemimodule, for every $i = 1, 2, \ldots, n$, it is obtained that $a + b \in \overline{\text{Apr}}\ S_i$, for every $i = 1, 2, \ldots, n$ and hence we have $a + b \in \bigcap_{i=1}^{n} \overline{\text{Apr}}\ S_i$. Now, since $\bigcap_{i=1}^{n} \overline{\text{Apr}}\ S_i = \overline{\text{Apr}}\ \bigcap_{i=1}^{n} S_i$, we have $a + b \in \overline{\text{Apr}}\ \bigcap_{i=1}^{n} S_i$.
2. Given arbitrary $r \in X$, $a \in \bigcap_{i=1}^{n} S_i$. By assumption, $S_1, S_2, \ldots, S_n$ are rough subsemimodules of Y over rough semiring X. Therefore, we obtain $a \in \bigcap_{i=1}^{n} S_i \subseteq S_i$, for every $i = 1, 2, \ldots, n$. Since S_i is a rough subsemimodule, for every $i = 1, 2, \ldots, n$, it is obtained that $ra \in \overline{\text{Apr}}\ S_i$, for every $i = 1, 2, \ldots, n$ and hence we have $ra \in \bigcap_{i=1}^{n} \overline{\text{Apr}}\ S_i$. Now, since $\bigcap_{i=1}^{n} \overline{\text{Apr}}\ S_i = \overline{\text{Apr}}\ \bigcap_{i=1}^{n} S_i$, we have $ra \in \overline{\text{Apr}}\ \bigcap_{i=1}^{n} S_i$.

Therefore, from 1 and 2 it is proven that $\bigcap_{i=1}^{n} S_i$ are rough subsemimodules of Y. ■

The following is an example to illustrate Proposition 2 by taking $n = 2$.

Example 4. Given a non-empty set $S \subseteq Y$ with $S' = \{\bar{0}, \bar{1}, \bar{5}, \bar{8}, \overline{10}, \overline{15}\}$ and $S'' = \{\bar{0}, \bar{2}, \bar{5}, \bar{9}, \overline{10}, \overline{19}\}$, to obtain $(S' \cap S'') = \{\bar{0}, \overline{10}\}$. $\underline{\text{Apr}}\ (S' \cap S'') = \{x \mid [x]_R \subseteq X\} = \varnothing$; $\overline{\text{Apr}}\ (S' \cap S'') = V_1 \cup V_6 = \{\bar{0}, \bar{4}, \bar{6}, \overline{10}, \overline{12}, \overline{16}, \overline{18}\}$.

Now, we will show that $S' \cap S''$ is a rough subsemimodule of a rough semimodule Y that satisfies the following axioms:

1. Based on Table 7, we have $a +_{20} b \in \overline{\text{Apr}}(S' \cap S'')$, for every $a, b \in S' \cap S''$.

Table 7. Cayley Table for $+_{20}$ in $\overline{Apr\ (S' \cap S'')}$

$+_{20}$	$\bar{0}$	$\bar{0}$	$\overline{10}$
$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$
$\overline{10}$	$\overline{10}$	$\overline{10}$	$\bar{0}$

2. Table 8 shows that $u \cdot_{20} a \in \overline{\text{Apr}}(S' \cap S'')$, for every $u \in X$ and $a \in S' \cap S''$.

Table 8. Cayley Table for Scalar Multiplication in $\overline{Apr\ (S' \cap S'')}$

$\cdot_{20}$	$\bar{0}$	$\bar{1}$	$\bar{6}$	$\bar{9}$	$\overline{10}$	$\overline{11}$	$\overline{14}$	$\overline{19}$
$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$
$\overline{10}$	$\bar{0}$	$\overline{10}$	$\overline{10}$	$\bar{0}$	$\bar{0}$	$\overline{10}$	$\bar{0}$	$\overline{10}$

Based on axioms 1 and 2 that have been satisfied, it can be concluded that $S' \cap S''$ are rough subsemimodules of rough semimodule Y over rough semiring X.

4. CONCLUSIONS

A rough semimodule over a rough semiring can be constructed using an approximation space $(\mho, \mu)$ where $\mho$ is a universal set and μ is an equivalence relation on $\mho$. Given an approximation space $(\mho, \mu)$ and Y is a rough semimodule of rough semiring X in $\mho$. If $S \subseteq Y$ with $\overline{\text{Apr}}(S) = T$, where T is a submodule of Y, then S is a rough subsemimodule of Y. Furthermore, if $S_1, S_2, \ldots, S_n$ are rough subsemimodules of Y over rough semiring X, then $\bigcap_{i=1}^{n} S_i$ is also a rough subsemimodule of rough semimodule Y if $\bigcap_{i=1}^{n} \overline{\text{Apr}}\ S_i = \overline{\text{Apr}}\left(\bigcap_{i=1}^{n} S_i\right)$.

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